Pedaling, Fast and Slow

The Race Towards an Optimized Power Strategy

Steven DiSilvio, Anthony Ozerov, Leon Zhou

Problem

Given:

- Cyclist (mass, power curve)
- Track

Optimize:

- Race strategy

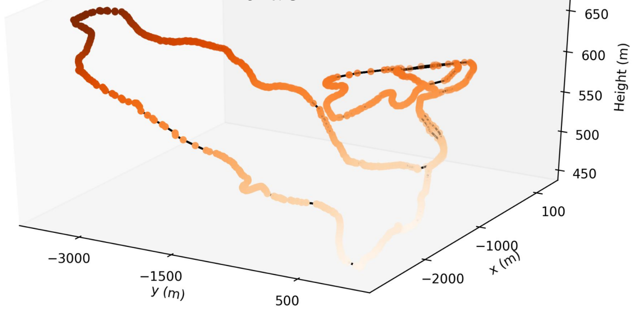
To minimize:

- Time to finish race

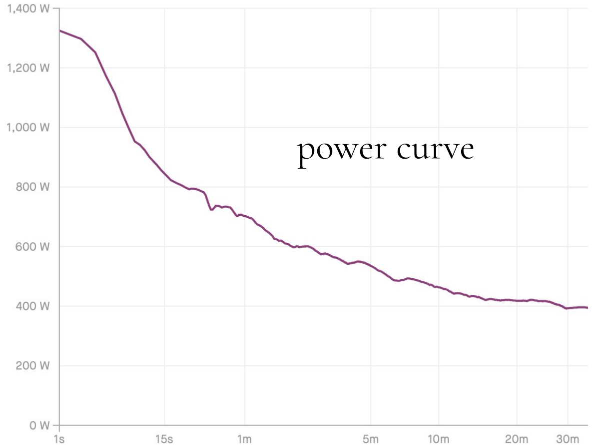
Subject to:

- Fatigue
- Physical constraints

track

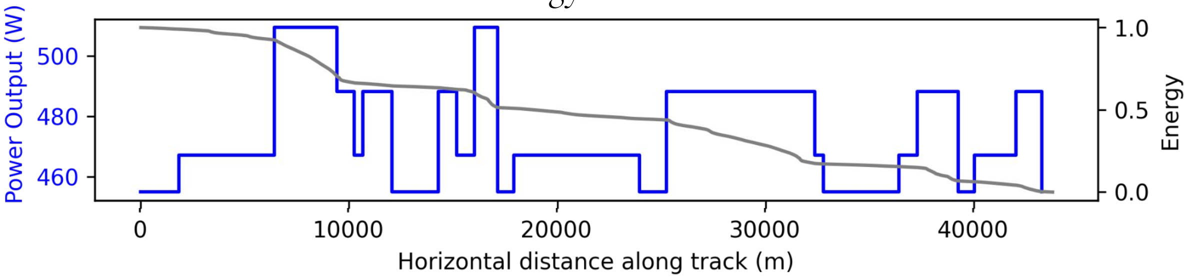


power curve



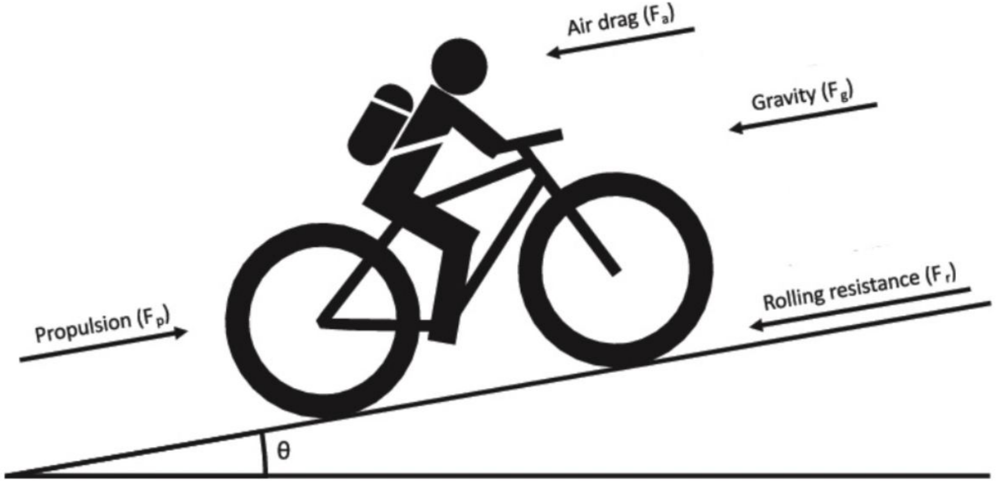
optimization

strategy



f : [**strategy**, track data, cyclist data] → race time ???

$$F_a = \frac{1}{2} C_d A v_a^2$$
$$F_g = mg \sin\left(\theta(x_h)\right)$$
$$F_r = \mu_r mg \cos\left(\theta(x_h)\right)$$
$$F_p = \frac{P(x)}{v}$$
$$F = F_p - F_a - F_g - F_r$$
$$\frac{dv}{dt} = \frac{F}{m}$$
$$\frac{dx}{dt} = v$$



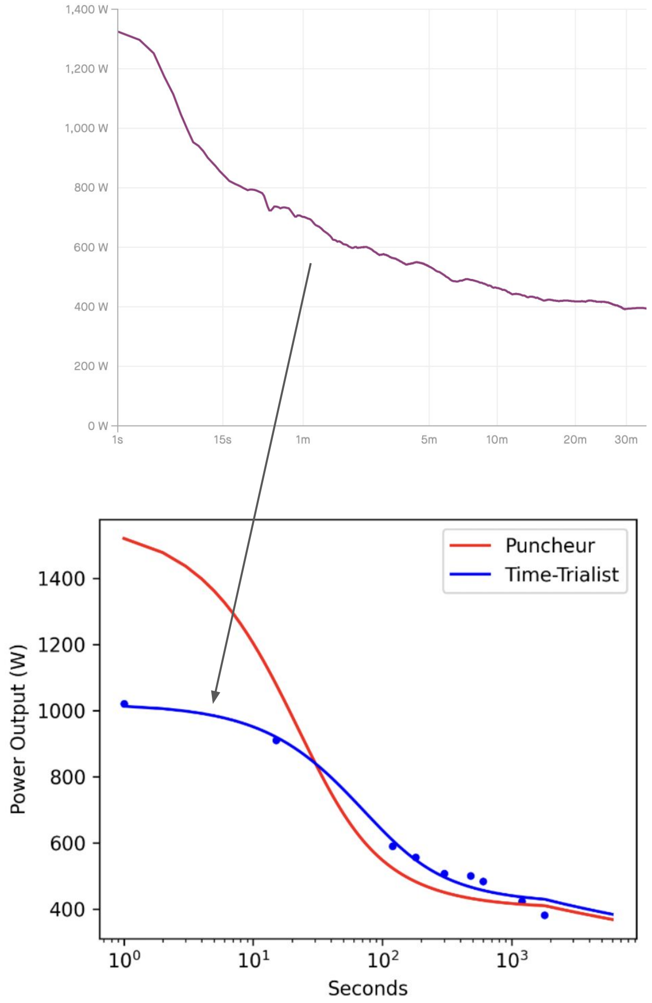
f : Euler approximate x and v together until x=end of race, and output time

Omni-PD Model

- Non-linear least squares to fit parameters
- Power levels were used as choices for the rider

Variable	Description	Units
P_{max}	Max Power	W
P_C	Critical Power	W
W'	Work above P_C (Anaerobic Work Capacity)	W
t	Time	s
T_{cpmax}	Time sustained at P_C	s
Constant	**Description**	
β	Linear Constant	-

$$f(t) = \begin{cases} \frac{W'}{t} * (1 - e^{t * \frac{P_{max} - P_C}{W'}}) + P_C & t \leq T_{cpmax} \\ \frac{W'}{t} * (1 - e^{t * \frac{P_{max} - P_c}{W'}}) + P_C - \beta * ln(\frac{t}{T_{cpmax}}) & t \geq T_{cpmax} \end{cases}$$



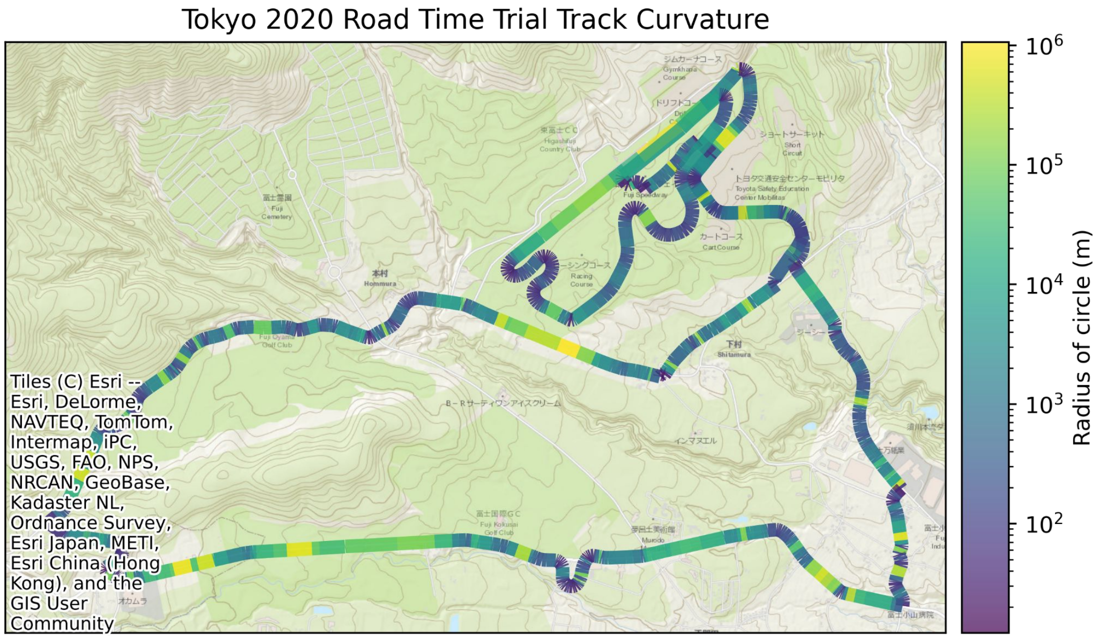
Fatigue Constraint

Inverse of power curve

$$\frac{dE}{dt} = \begin{cases} \frac{-1}{f^{-1}(P(x_h))} & \text{if } P(x_h) > P_C \\ \frac{1}{7200 P_C}(P_C - P(x_h)) & \text{if } P(x_h) \leq P_C \end{cases}$$

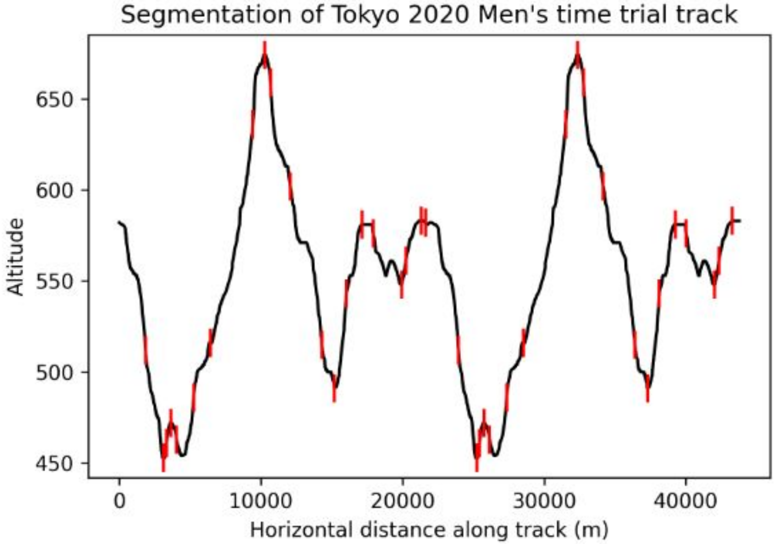
E Euler approximated along with x and v, power output is capped at a low, sustainable value when E reaches zero

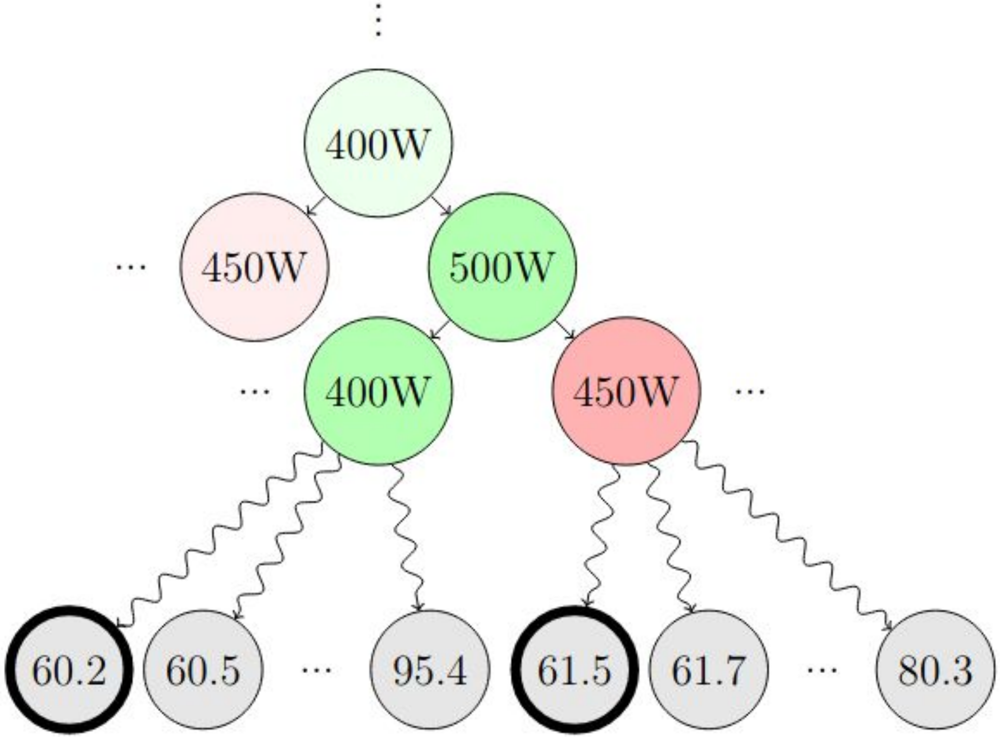
Physical Constraint

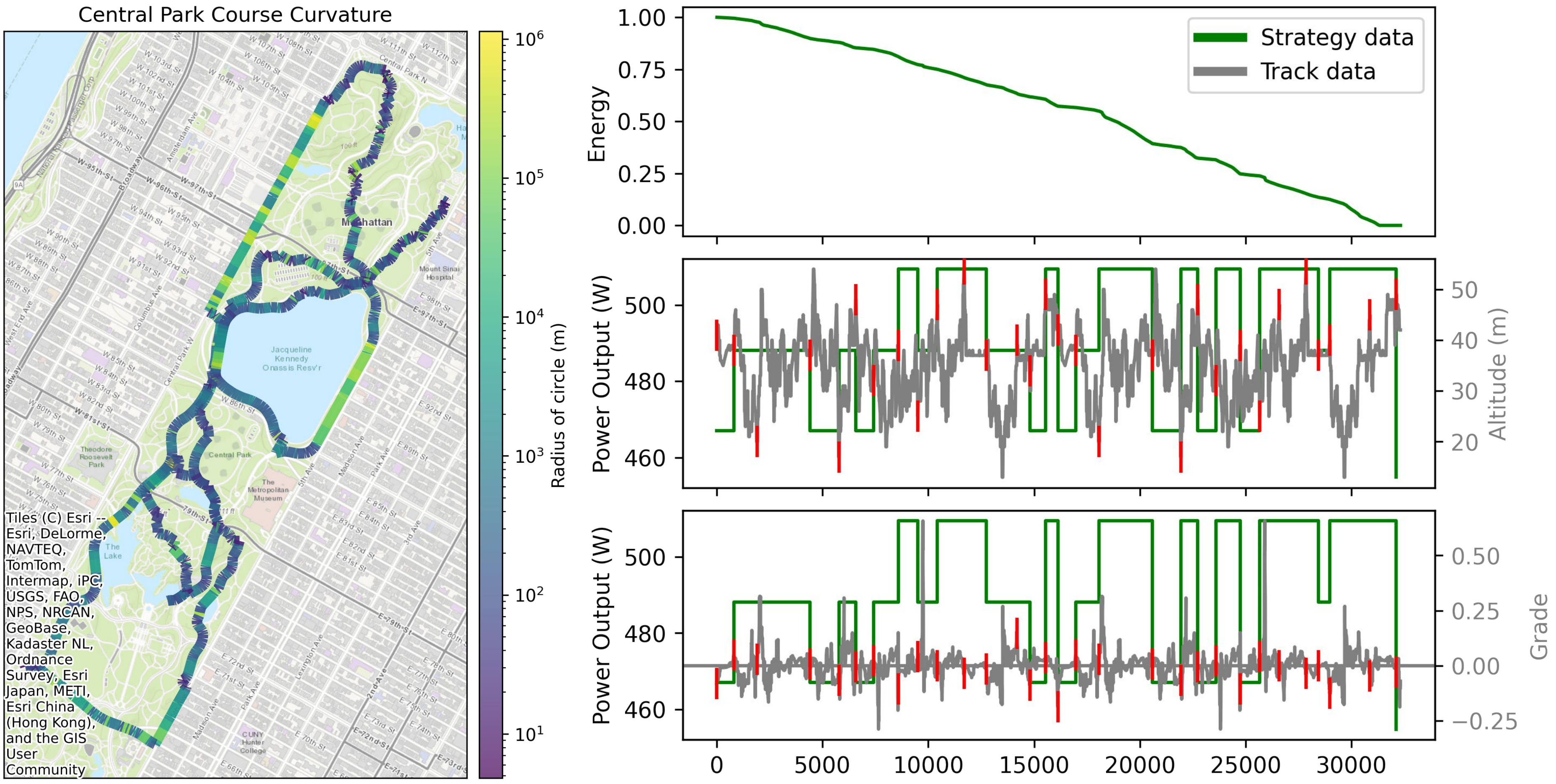


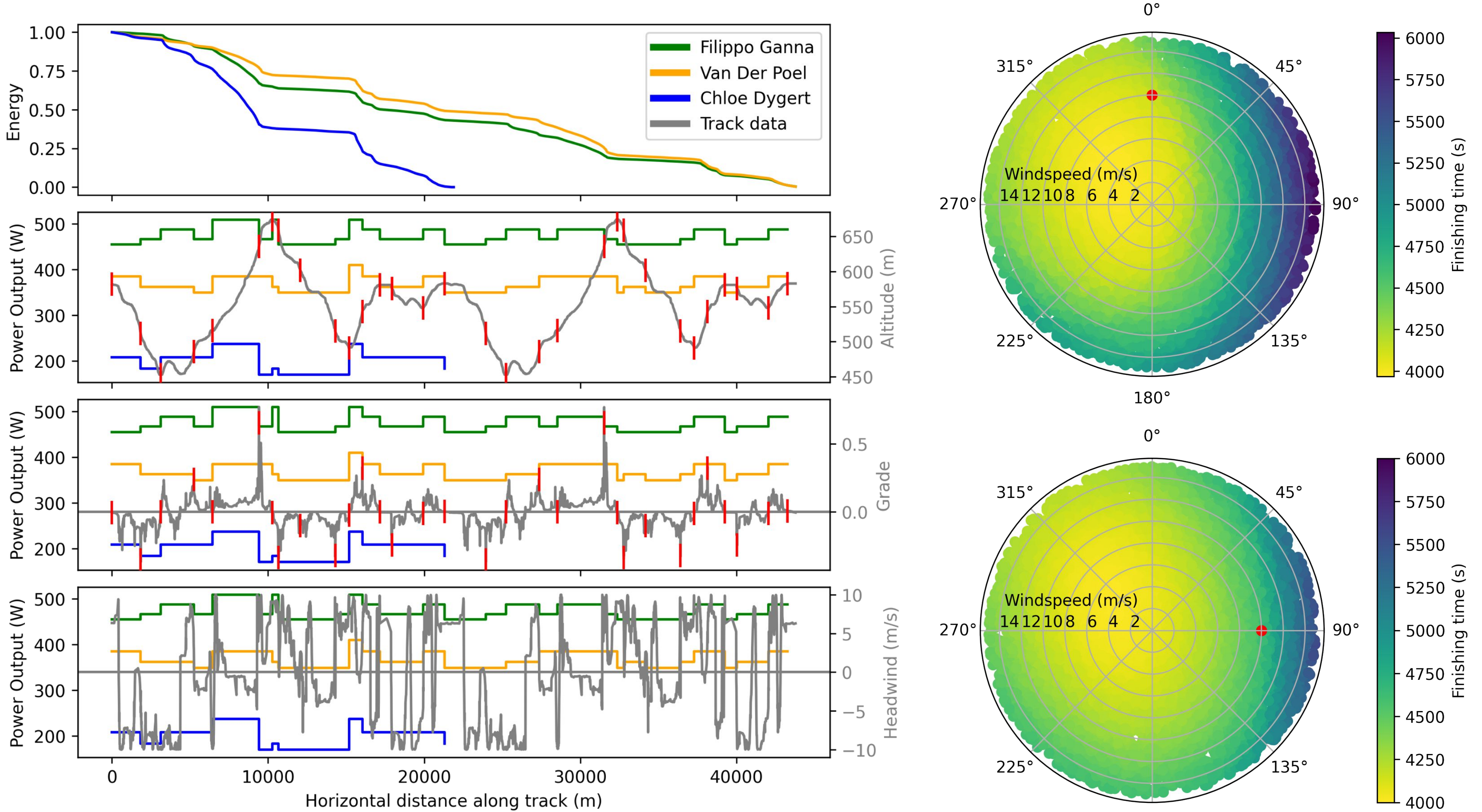
v is capped at the maximum speed around a curve

Optimizing strategy to minimize f






Central Park Course Curvature
Radius of circle (m)
10^1
10^2
10^3
10^4
10^5
10^6
Tiles (C) Esri -- Esri, DeLorme, NAVTEQ, TomTom, Intermap, iPC, USGS, FAO, NPS, NRCAN, GeoBase, Kadaster NL, Ordnance Survey, Esri Japan, METI, Esri China (Hong Kong), and the GIS User Community
Jacqueline Kennedy Onassis Resv'r
Central Park
The Lake
The Metropolitan Museum
Manhattan
Energy
1.00
0.75
0.50
0.25
0.00
Strategy data
Track data
Power Output (W)
Altitude (m)
Grade
460
480
500
20
30
40
50
−0.25
0.00
0.25
0.50
0
5000
10000
15000
20000
25000
30000



Filippo Ganna
Van Der Poel
Chloe Dygert
Track data
Energy
Power Output (W)
Altitude (m)
Grade
Headwind (m/s)
Horizontal distance along track (m)
Windspeed (m/s)
14 12 10 8 6 4 2
0°
45°
90°
135°
180°
225°
270°
315°
Finishing time (s)

Paper:

https://github.com/anthonyozerov/optimal-cycling/blob/main/2022_mcm_submission.pdf

Code:

https://github.com/anthonyozerov/optimal-cycling