Introduction of 3 Fictional Dragons to a Non-Fictional Planet

John Blackwelder, Steven DiSilvio, and Anthony Ozerov

Abstract

We capture the effect of the introduction of dragons to Earth using 3 models. Effects on local ecosystems are simulated by two competing models: a differential equation model, and an agent-based model. We find that dragons do not destroy their local ecosystems. Based on this result, we model the migratory behaviour of dragons and their effects on the planet, and find that dragons settle down once they find a suitable climate.

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1 Introduction

As a result of their immense destructive powers and all but impenetrable armor, the dragons depicted in the television series *Game of Thrones*[3] play pivotal roles as the story unwinds.

However, besides the occasional shot of a dragon incinerating livestock or leaving people in awe as it flies overhead, the show does little to portray the quotidian habits of a dragon and the true scale of their ecological footprint in Westeros.

Fortunately, several key traits are known. Dragons undoubtedly face no competition from other animals living in Westeros, giving them clear cut apex predator status. According to Tyrion Lannister, a well read and knowledgeable character on the show, dragons are extremely intelligent, maybe even to a degree surpassing humans. The three living dragons depicted in the show appear on the order of magnitude of an enormous airplane, but even they are considerably smaller than Balerion the Black Dread, a legendary dragon whose skull appears several times on the show, was during its lifetime.

At this point it becomes obvious that a dragon's size and abilities make it incomparable with any animal living on earth today. Thus, the degree to which a dragon would impact earth's environment is an intriguing question. We construct three models in order to determine what sort of effect introducing 3 dragons to the environment would have. We start with a **preliminary regional agent-based model**, which is meant to broadly determine how dragons impact surrounding fauna. Next, we construct a **regional differential equation model** in order to corroborate our preliminary results and determine the impact of a single dragon on local flora. Finally, our **global migration model** puts our previous results in context, as we seek to analyze dragon movement patterns and how they relate to environmental conditions.

Not only is the introduction of dragons to our world an interesting thought experiment, but comparing our results with the very real environmental impact humans have had on the world for millennia provides an entirely different lens through which to assess our own destructive behavior. We hope that our work leads people to reflect on the fragility of our environment and the role that we all play in its continuing degradation.

2 Problem Statement

We study the behavior of three dragons from George R.R. Martin's *Game of Thrones* when brought to Earth, and the management strategies most suited to them as a result. Within a single ecosystem, we analyze dragon's diet and ecosystem impact. We also study dragon migratory behaviour between ecosystems of different climates. We assume that dragons are cold-blooded reptiles who can fly far, are apex predators, yet do not intrinsically seek conflict. We assume that dragons are born at 10kg and grow to 30-40kg by their first birthday. We further assume that the dragons, modeled on Martin's, reproduce on a timescale so great that modeling their reproduction would not be relevant on a human timescale.

3 Modeling Methodology

3.1 Regional Agent-Based Model

Motivation The primary purpose of this model is to model predator-prey interactions within the local ecosystem of a single dragon's territory by considering the actions that it takes in hunting as well as the reproductive practices of prey at an individual level. Tracking the actions of these individual agents via a daily time-step over the course of several years will allow the weight of the dragon and the population of deer to be determined over time, giving insight into the sustainability of dragons in the wild as well as their impact on surrounding prey populations.

Mapping The model is situated within a 100km by 100km continuous square region representing the dragon's territory.

Model Initialization We randomly situate a single dragon as well as 50,000 deer within the dragon territory. We consider deer to be the archetypal dragon prey species, since they are geographically ubiquitous and moderately sized. The choice of 50,000 deer in our region corresponds to a reasonable starting population density of 5 deer/km^2.

In an effort to make this model as generally applicable as possible, we do not assign any additional parameters to the region itself. However, we do assign our dragon a weight of 10,000 kgs and an r value of .5 km, representing the radius of the total area in which a dragon can hunt in one sitting before having to relocate (more on this later).

Additionally, the model is assumed to begin on the first day of the year in January, and being that this is within a month of the end of the typical mating season for deer, roughly a quarter of all does are initialized as being pregnant.

Dragon Weight and Metabolic Characteristics The dragon's weight is tracked considered over time, with the weight change of a dragon calculated at the end of each day via the following equation:

$$\Delta w(t) = \frac{e_d(t) - e_m(t) - e_f(t)}{7,000}$$

where e_d, e_m and e_f represent the calories consumed via deer, burned via metabolic processes and burned through flight activity respectively, each given by the following equations:

$$e_d(t) = 119,040\,(d_{total}(t))$$

$$e_m(t) = 139\,(w(t))^{(0.889)}$$

$$e_f = 119.04\,(D(t))\,\sqrt{w(t)}$$

where $d_{total}(t)$ represents the total deer eaten on day t, $w(t)$ is the weight of the dragon on day t, and $D(t)$ is the total distance travelled by the dragon on day t.

Hunting Conditions Dragons are assumed to hunt on a daily basis by flying roughly 50 km distances daily, stopping periodically throughout their flight at random intervals to land and hunt deer where it sees fit. Though no specific environment is taken into account, we assume that dragons must land in order to hunt prey, which would be consistent with environments having large canopy coverage from trees that would otherwise prevent a dragon's ability to hunt solely from the air. When a dragon lands, any deer within .5 km of the dragon are susceptible to being eaten, with the amount of which the dragon eats given by the following function:

$$f(x) = \begin{cases} d_e & \text{if } x \geq 3d_e \\ \left\lfloor \frac{x}{3} \right\rfloor & \text{if } x < 0 \end{cases}$$

where x is the amount of deer within the .5 km radius and d_e is the minimum amount of deer a dragon needs to consume on a particular day to surpass its metabolic expenditure (more on this later). Thus, in a single sitting a dragon never eats more than d_e nor a third of the deer in its very immediate vicinity.

Deer Reproduction Deer are assumed to reproduce seasonally, with mating season occurring between mid-October and mid-December, and have a gestation period lasting 180 days. In our model, roughly a quarter of all female deer will reproduce every year, and each female gives birth on the same day in spring, raising their offspring until 10 days before mating season. Each female has an equal likelihood of producing one offspring as two for a given pregnancy, and during the time period in which a female is looking after her young she is assigned a caloric value one tenth that of an adult deer for each fawn it is raising. When deer become fully matured after roughy 180 days, they separate from their mother and have a normal caloric value.

3.2 Regional Differential Equation Model

Motivation Differential equation models are commonly used to model predator-prey interactions, describing the number of predators and the number of prey. But with the release of 3 dragons who grow from very small to very large and reproduce on a long timescale it becomes more important to describe the weight of a predator. Thus, we model the interactions of 1 dragon with an ecosystem consisting in the dragon, prey which the dragon eats, and vegetation which the prey live off of and which the dragon burns. Prey is taken to be deer, which are our prototypical example of the kind of large land animals that a dragon would eat.

Dragon weight and growth We assume that dragons, like some other large reptiles, have indeterminate growth. Thus, their growth is not determined by age, but on their net energy intake. Equation (1) describes their growth:

$$\frac{dw}{dt} = \frac{f(t) - e(t)}{r} \tag{1}$$

Where f is calorie intake, e is the dragon's energy requirement, and r is the number of calories needed for 1kg of growth. When $f - e < 0$, the dragon is set to not grow.

Energy requirements We split the yearly energy requirements of the dragon into two parts: a basal metabolic rate, and an active energy requirement:

$$e(t) = e_m(t) + e_a(t)$$

Where e_m is the basal metabolic rate and e_a is the active energy requirement.

Basal metabolic rate The basal metabolic rate is calculated using:

$$e_m(t) = 139.065(w(t)^{0.889}) \tag{2}$$

Which is based on the basal metabolic rate observed in the Komodo Dragon, another large reptile [10]. This does not depend on climate as, being cold-blooded, dragons do not need to expend energy to attain a certain body temperature.

Active energy requirement To get from prey to prey, dragons need to fly. This flight takes energy, based on the distance to the prey and the weight of the dragon. The following equation describes this relationship:

$$e_a(t) = d(t)w(t)e_f \cdot \frac{f(t)}{k}$$

Where $d(t)$ is the average approximated distance between deer, e_f is a constant describing how much energy per kg of mass it takes to fly 1km, and $\frac{f(t)}{k}$ is the dragon's deer consumption.$d(t)$ is estimated using the following approximation:

$$d(t) = 2\sqrt{\frac{A}{2\pi p(t)}}$$

Where A is the area of the ecosystem and $p(t)$ is the number of prey. This approximation works by assuming that the deer are uniformly distributed through the area, then equal-sized circles are constructed around every deer to fill the area, and $d(t)$, the diameter of the circles, is set so that the combined area of the circles is equal to the area of the ecosystem. This approximation will be off by a constant factor, but what it captures is the inverse square root relationship between deer density and average distance between deer.

Food intake When there is a plethora of deer in a dragon's hunting range, its deer consumption is ultimately limited by how long it takes for it to attack, eat, and digest a deer. In the field of Ecology, this is commonly represented using a Type II functional response curve, which relates prey consumption to prey density. Different types of functional responses are illustrated in Figure 1. Following Holling [7], we use the following equation to describe food intake:

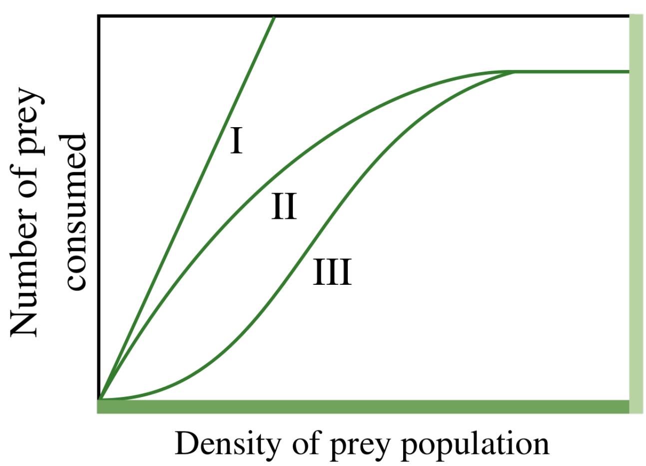

Figure 1: Comparison of different functional responses[5]

$$f(t) = kw(t)\frac{\frac{ap(t)}{A}}{1 + \frac{ahp(t)}{A}}$$

Where k is the number of calories in a deer, $w(t)$ is the mass of the dragon, a is the attack rate of a dragon (since we assume that dragons are excellent hunters, we take it as the area a dragon is able to search), h is the handling time of a dragon (how long it takes to eat 1 deer), $p(t)$ is the number of prey, and A is the area of the region. Note the one difference with Holling's Type II model: we care about how the dragon's size affects its interactions, so we think of the Type II functional response as happening for every kilogram of the dragon, so a and h are really defined per kilogram of the dragon, hence we multiply the result by the dragon's weight.

Number of prey We assume that the deer take a logistic growth curve, with a maximum controlled by the vegetation level. However, deer must also be removed from the system as the dragon eats them:

$$\frac{dp}{dt} = jp(t)\left(1 - \frac{p(t)}{p_{cap}(t)}\right) - \frac{f(t)}{k}$$

Where j is the deer growth rate and p_{cap} is the current carrying capacity for deer, assumed to be the product of the vegetation density v and the maximum possible carrying capacity p_{max}, *i.e.*

$$p_{cap}(t) = v(t)p_{max}$$

Vegetation density Vegetation levels also follow a logistic growth curve, but with some vegetation being removed as the dragon burns it:

$$\frac{dv}{dt} = Alv(t)(1 - v(t)) - \frac{gw(t)v(t)}{A} \tag{3}$$

Where l is the vegetation growth rate, v is the current vegetation level, and g is the dragon destructiveness per weight per vegetation.

Parameters Parameters, their units, and their values, are identified in Table 1. They were chosen based on the problem requirements, the purpose of the model, and real-world data.

3.3 Global Migration Model

Motivation Once small-scale predator-prey interactions are understood, it becomes important to understand how the three dragons will interact with the world at large. The key aspect of this is their migration patterns, which the regional models are not built to capture. We propose a continuous-space migration model that combines data-based climatic factors with any given belief about dragons' level of ecosystem disruption to simulate how they move across the surface of the Earth.

Dragon Movement Every day of the simulation, each dragon checks if the suitability at its location on that day, $s(t, x, y)$ is below a value s_{min}, chosen to be -0.5 to make the dragons fickle. If the suitability is low, the dragon will look a random distance less than 4 degrees away in 4 randomly chosen directions (one somewhere to the upper right, one somewhere to the lower right. . .), and choose to migrate to the point on land with the greatest suitability (or stay where it is, if no point has greater suitability).

Table 1: Differential equation model variables and constants

Variable	Description	Range	Units	
w	Dragon weight	$[10, \infty)$	kg	
p	Prey number	$[0, p_{cap}]$	deer	
v	Vegetation level	$[0, 1]$		
f	Food intake	$[0, \infty)$	kcal yr^{-1}	
e_t	Total energy requirement	$[0, \infty)$	kcal yr^{-1}	
e_m	Basal Metabolic Rate	$[0, \infty)$	kcal yr^{-1}	
e_a	Active energy requirement	$[0, \infty)$	kcal yr^{-1}	
d	Average distance between deer	$[0, \infty)$	km	
p_{cap}	Prey carrying capacity	$[0, \infty)$	deer	
Constant	Description	Value	Units	Source/rationale
a	Attack rate, or search efficiency	3,650	km^2 yr^{-1} kg^{-1}	Chosen so that a 10kg dragon can search 100km^2 per day
h	Handling time	2.7	yr deer^{-1} kg^{-1}	Based on snake satiety times, adjusting for prey size [11]
r	Calories needed for 1kg of growth	6,000	kcal kg^{-1}	Based on the calorie content of reptiles [12]
A	Area of ecosystem	2,000	km^2	The size of a large nature preserve
j	Deer reproduction rate	0.25	–	Chosen to be low
l	Vegetation production rate	1	–	Chosen to be high
w_0	Initial dragon mass	10	kg	Per problem statement
p_0	Initial deer number	40,000	deer	Creates a deer density of 20deerkm^{-2}
v_0	Initial vegetation density	1	–	Assuming ecosystem starts at carrying capacity
p_{max}	Maximum prey carrying capacity	40,000	deer	Assuming ecosystem starts at carrying capacity
g	Dragon destructiveness	100	km^2 yr^{-1}	Chosen to be high
k	Caloric content of a deer	119,040	kcal	Real-world data [4]
e_f	Dragon flight energy	$k/1000$	kcal km^{-1} kg^{-1}	Chosen so that a 1000kg dragon would need to travel no more than 1km between deer in order to recoup costs

Ecosystem Destruction Each day of the simulation, each dragon will incur D damage to its ecosystem, where D can be chosen to be 0 if we believe that dragons can integrate into an ecosystem, or a higher value if we believe they incur long-term damage.

Suitability The dragons, in migrating, seek optimal living conditions. The quality of the local conditions at any point are defined by the following equation:

$$s(t, x, y) = s_t(t, x, y) - d(t, x, y) \tag{4}$$

Where $s(t, x, y)$ is the local suitability on day t at longitude x and latitude y, $s_t(t, x, y)$ is the suitability of the local climate under those same conditions, and $d(t, x, y)$ is a negative modifier based on equal to the sum of previous local ecological destruction. $s_t(t, x, y)$ and $d(t, x, y)$ data both have a resolution of 2.5°.

Climate suitability Reptiles, being ectotherms, do not regulate their internal temperature themselves. Instead, they use ambient temperature and solar radiation to reach their desired temperature. $s_t(t, x, y)$ is calculated using the following equation:

$$s_t(t, x, y) = -(T(t, x, y) + 5I(t, y) - c_{opt})^2 \tag{5}$$

$T(t, x, y)$ is the surface temperature in Celsius on day t of the 2020 at longitude x and latitude y, taken from the NCEP surface temperature reanalysis dataset for 2020 [8]. $I(t, y)$ is the solar irradiance as a proportion of maximum possible solar irradiance, and c_{opt} is the optimal temperature for dragons. We assume that c_{opt} is 30°C, and that radiation from the sun can, at most, increase the effective temperature for dragons by 5°C. The idea here is that reptiles need to live in an optimal climate to maintain their body temperature, but they are also able to bask in the sun to increase their body temperature. Dragons, being large, flying animals, are assumed to not be able to hide from the sun, so the irradiance must always be taken into account. Forming this into a downwards-opening quadratic causes a peak suitability when dragons are at their optimal temperature, with slowly and then rapidly decreasing suitability as the temperature leaves that range.

Solar Irradiance Irradiance is calculated as follows:

$$I(t, y) = \cos(y - l(t))$$
$$l(t) = -23.4 \cos\left(\frac{t}{365} \cdot \frac{180}{\pi}\right)$$

y is the latitude of the dragon, $l(t, x, y)$ is the latitude at which solar irradiance is maximum, and t is the number of days since the simulation–which starts in winter–started. The cosine function that defines $l(t)$ causes the latitude at which solar irradiance is at maximum to shift from 23.4°S around December, to 23.4°N around June, mimicking the yearly variation in solar zenith angle. The difference between that latitude and y is then used to calculate the solar irradiance at latitude y.

4 Model Results

4.1 Regional Agent-Based Model

Our regional agent-based model produced the result that the dragon would die within roughly 8.5 years of its initial introduction into the local ecosystem, with the dragon reaching a maximum weight of roughly 58312 kg on the 38^{th} day, but eventually falling below the 5000 kg threshold which led to death. The final deer population at the time of death of the dragon was 6181, with the total after the conclusion of the 10 years being 6,980. Below in Figure 2 are the maps of the ecosystem taken exactly 1 year apart for the first 10 years following the initial introduction of the dragon into the ecosystem, with each black dot representing a deer and the red marker representing the location of the dragon. Table 2 represents these values explicitly:

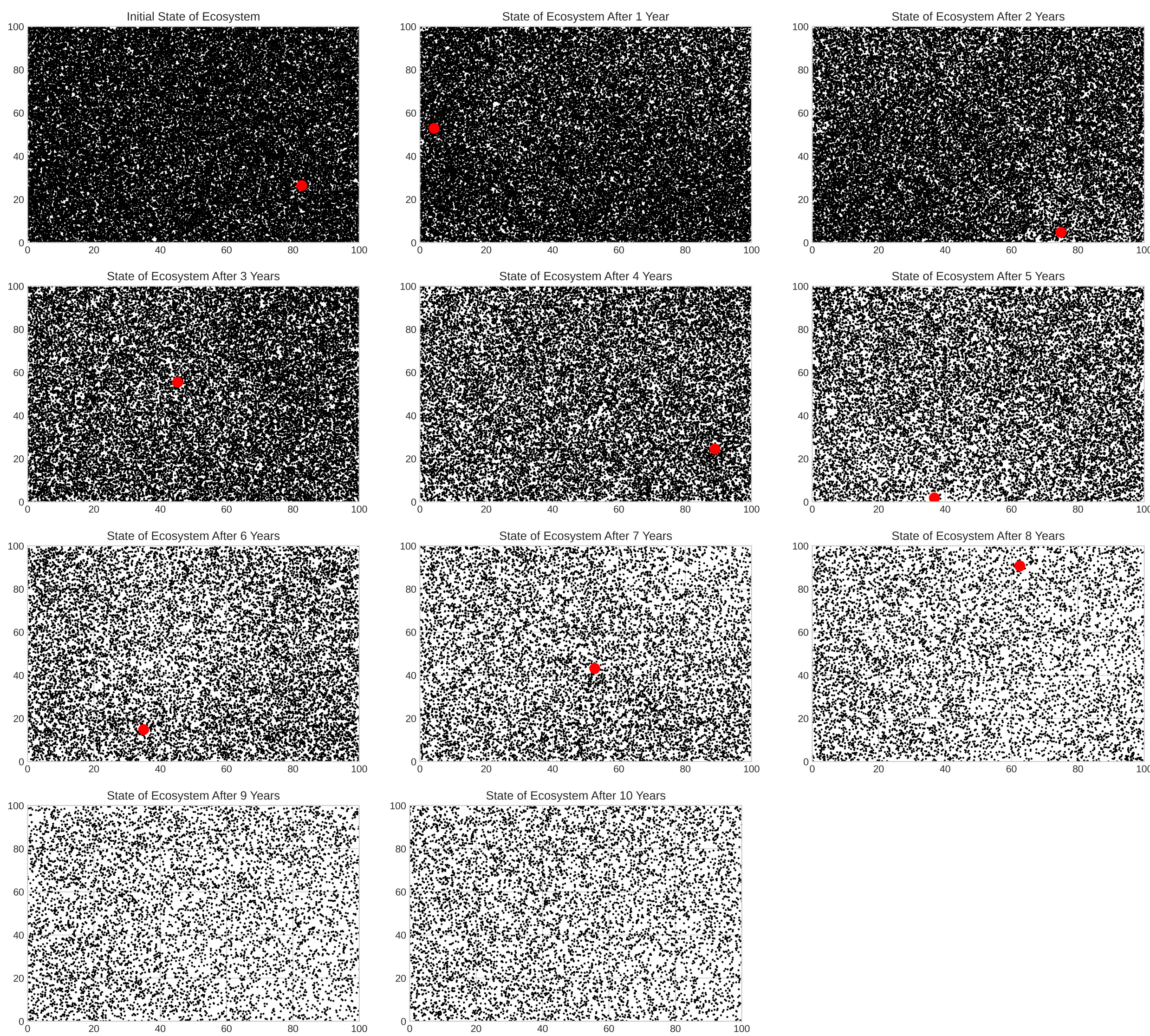

Figure 2: Ecosystem maps over the course of 10 years following dragon introduction

Table 2: Deer Population and Dragon Size Over the Course of 10 Years in the Regional Agent-Based Model

Time Passed (years)	Deer Population	Dragon Size (kg)
0	50,000	10,000
1	41,909	51,349
2	34,926	36,087
3	28,437	31,656
4	22,720	29,626
5	17,920	18,534
6	13,662	14,455
7	10,231	11,696
8	7,393	8,495
9	6,576	N/A
10	6,980	N/A

4.2 Regional Differential Equation Model

The Regional Differential Equation Model was numerically solved using an Euler approximation.

4.2.1 Initial Results

Results under default parameters We see in Figure 3 that, rather than wiping out the deer population like the authors of the winning paper conclude, the system actually reaches a steady state. Dragon growth stops as soon as there is a decrease in deer density and an increase in mass making flight between deer expensive, leaving no energy for growth. The dragon thus reaches a natural maximum weight of around 2800kg within 4 years. Moreover, vegetation density and deer population reach a steady quantity. The dragon, by 5 years of age, consumes over 120 million calories, which are provided to it by the reproducing deer population.

4.2.2 Parameter Studies

Varying ecosystem area Holding the initial deer density constant, varying the area of the ecosystem illustrates how the dragon affects ecosystems of different sizes. In Figure 4, it is evident that small ecosystems with values of $A < 1,000$ are not able to support a dragon. Either the vegetation is burned up by the dragon, hence preventing deer reproduction and destroying the ecosystem, or the deer population is simply not enough to support the dragon's food requirements. Moreover, the size of the ecosystem, and, indirectly, the number of prey clearly affect the maximum size the dragon reaches. As the ecosystem size increases, the effect of the dragon's predation has a smaller impact on deer density, allowing the dragon to grow larger as it expends less energy on flight.

Varying prey carrying capacity In Figure 5, it is evident that, holding area constant, varying the prey carrying capacity (and, hence, the initial number of deer) affects the size of dragon an ecosystem is able to support. At a carrying capacity of 400,000 deer, which, in an area of 2,000km^2 equates to 200 deer per square kilometer, we see that the dragon is able to grow to an impressive 10,000kg. On the other hand, at an initial 10,000 deer, the dragon grows to a measly 1,300kg. A higher prey carrying capacity means a higher density of prey,

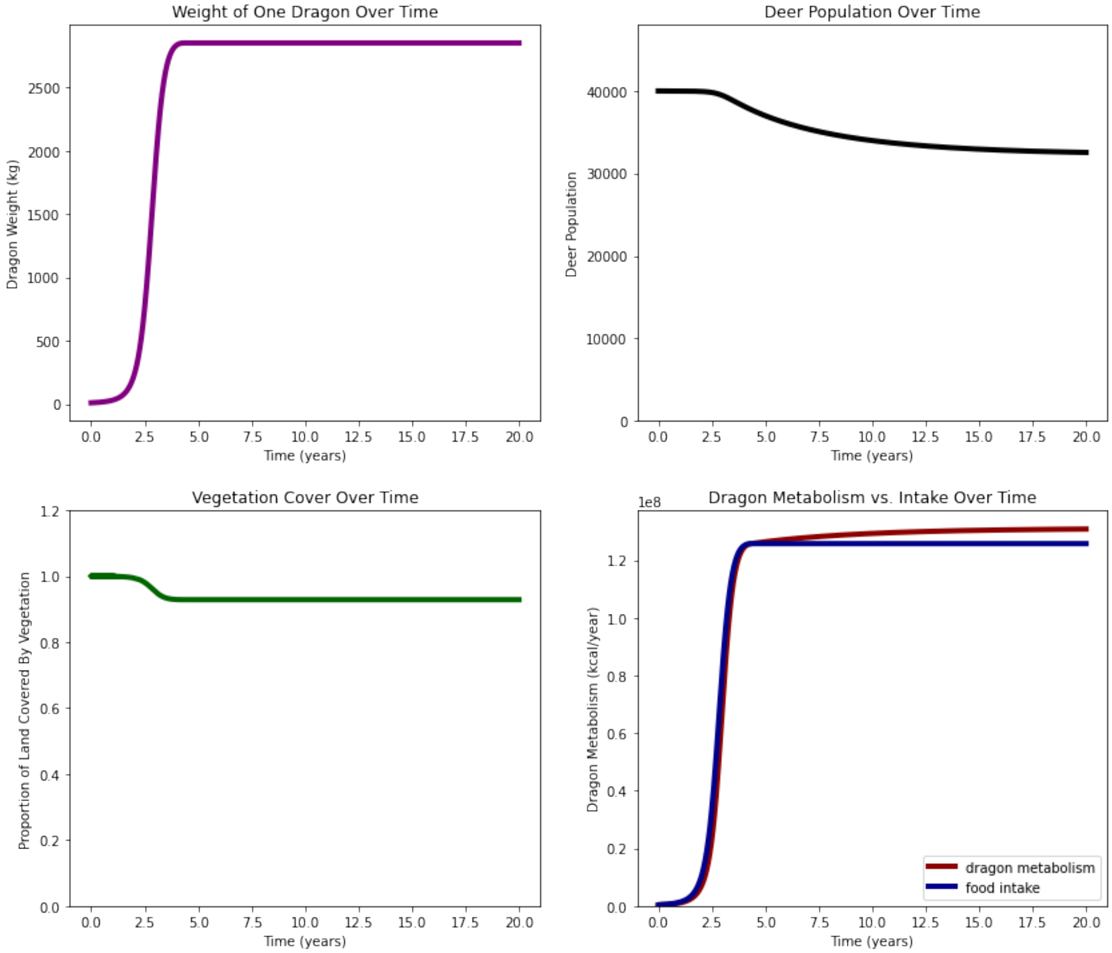

Figure 3: Population dynamics under the model parameters defined in Table 1

allowing the dragon to fly less to eat the same amount of food, hence gaining more energy and being able to reach a higher maximum weight.

4.3 Global Migration Model

4.3.1 Example Simulations

To illustrate the results of this model, the following examples may be observed.

No destruction When dragons are assumed to integrate into their ecosystems, *i.e.* when D is set to 0, we can, for example, observe the dragon movements in Figure 6. Under the conclusions of the differential equation model, which shows that a dragon is able to integrate itself into a sufficiently large ecosystem, we see that where a dragon finds a climate suitable year-round, it may settle there.

Destruction When dragons are assumed to destroy their ecosystems, *e.g.* when $D = 0.01$, we can, for example, observe the dragon movements in Figure 7. If dragons are found unable

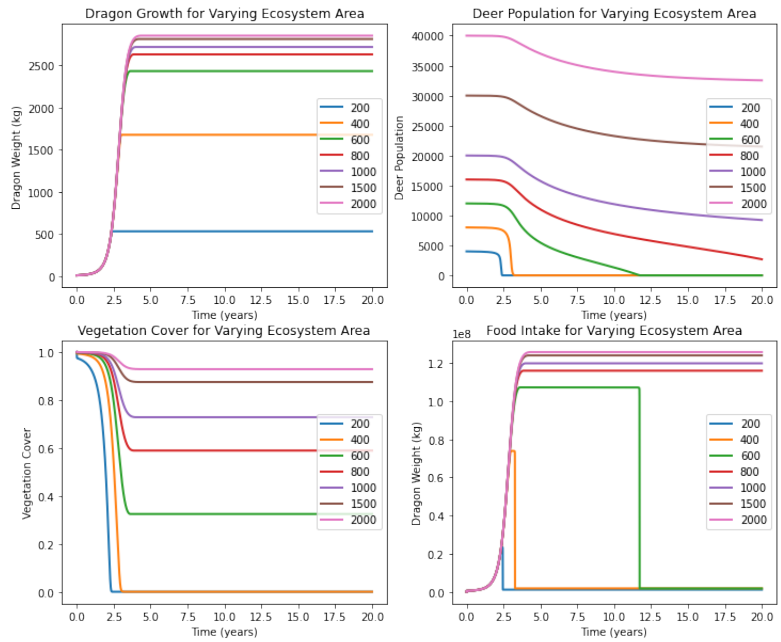

Figure 4: Population dynamics under varying ecosystem area

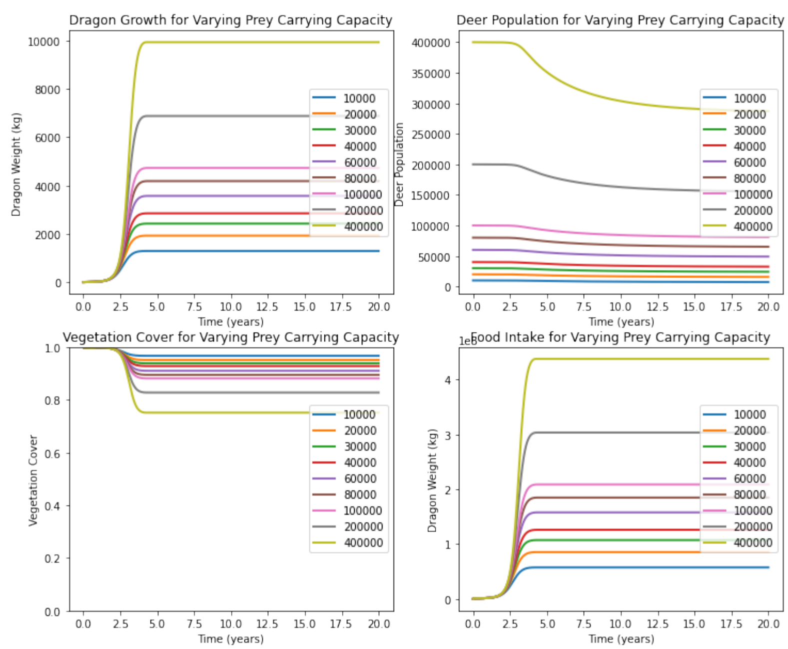

Figure 5: Population dynamics under varying prey carrying capacity

to integrate into ecosystems, we see that they migrate over a large area, causing widespread destruction.

Ecosystem destruction with multiple dragons When dragon destructiveness is set to 0.1, and three dragons are introduced, we may observe the total destruction caused in Figure 8. We see that the dragons disrupt ecosystems over a wide area, but particularly so

Figure 6: Migration with no ecosystem degradation. The dragon, starting near Moscow, slowly approached a more suitable climate and finally found one in Ethiopia, where it was able to stay as the ecosystem remained intact.

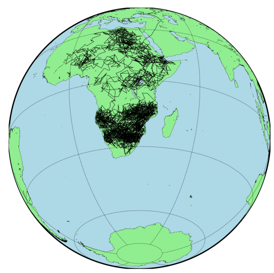

Figure 7: Migration with ecosystem degradation, $D = 0.1$. The dragon moves all over the continent of Africa, constantly needing to migrate to find untouched ecosystems.

in climates they prefer. Polar regions remain untouched.

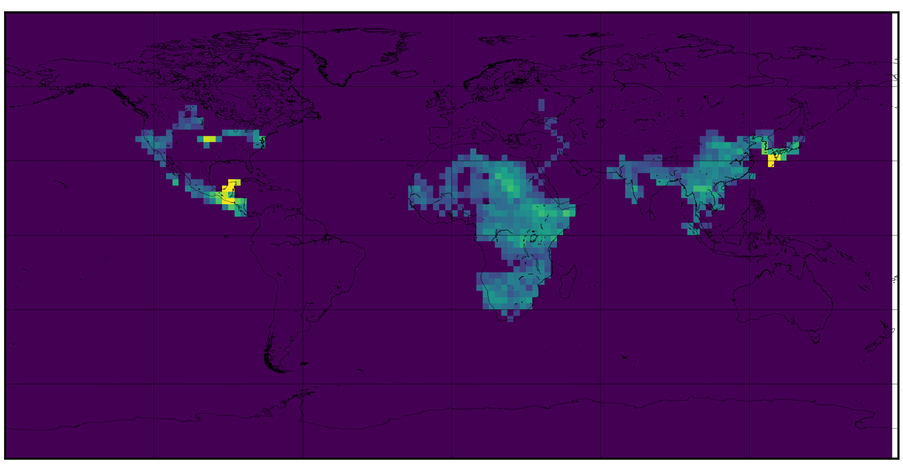

Figure 8: Total ecosystem destruction with 3 dragons, $D = 0.1$. The dragons cause destruction over a wide area, but increased destruction is found in the Yucatan peninsula and southern Japan.

4.3.2 Large Sample Analysis

Total migration distance Given the brief observations above, we may expect that, when $D = 0$, the total migration distance of every dragon is much lower than when $D = 0.1$.

Running the simulation multiple times confirms this belief. Consider Figure 9, which displays the frequencies of different migration distances under the two levels of D. It clearly shows that, when the dragons are assumed to effectively integrate into their ecosystems, they are not particularly inclined to migrate once they find the right climate. The median migration distance under $D = 0$ is but 14,400km in the first 1,000 days, whereas under $D = 0.1$ it is 121,100km. In the next 1,000 days, the difference is even more pronounced, with nondestructive dragons having already settled into a good climate and destructive dragons needing to scour more of the Earth in search of untouched ecosystems. In the second 1,000 days, the median migration distance under is 8,200km under $D = 0$ and 127,100km under $D = 0.1$.

Dragon migration distances at different levels of D

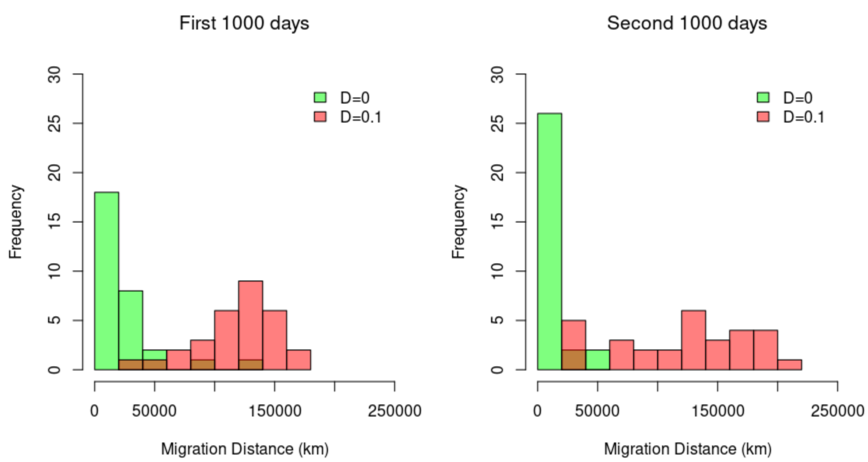

Figure 9: Histogram of dragon migration distances when $D = 0$ and $D = 0.1$. For each value of D, 10 simulations with 3 dragons each were run for 2,000 days. The frequency of dragon migration distances in the first 1,000 days is displayed on the left, and that in the second 1,000 days is displayed on the right. On the left, we see a clear separation in the migration distances of dragons under the two values of D in the first 1,000 days, with dragons migrating substantially less under $D = 0$. On the right, this trend is even more pronounced.

Dragon Settling Locations Figure 10 displays the positions of dragons after 2,000 days of the simulation. We see that dragons from simulations where $D = 0$ cluster around the equator in Africa and in the warm climates of Southeast Asia. On the other hand, dragons from simulations where $D = 0.1$ do not cluster as much in these regions. We believe this is a result of dragons destroying the ecosystems in the prime climates that they moved to initially, forcing them into suboptimal climates with intact ecosystems. In the long-run, the dragons may destroy ecosystems so extensively that they are pushed into climates wholly unsuitable for reptiles.

4.4 Result Synthesis and Application

4.4.1 Scenario 1: Dragon nature preserve

Given the result of the Regional Differential Equation Model that a large enough ecosystem can support a dragon, we consider the various properties of a nature preserve suitable to

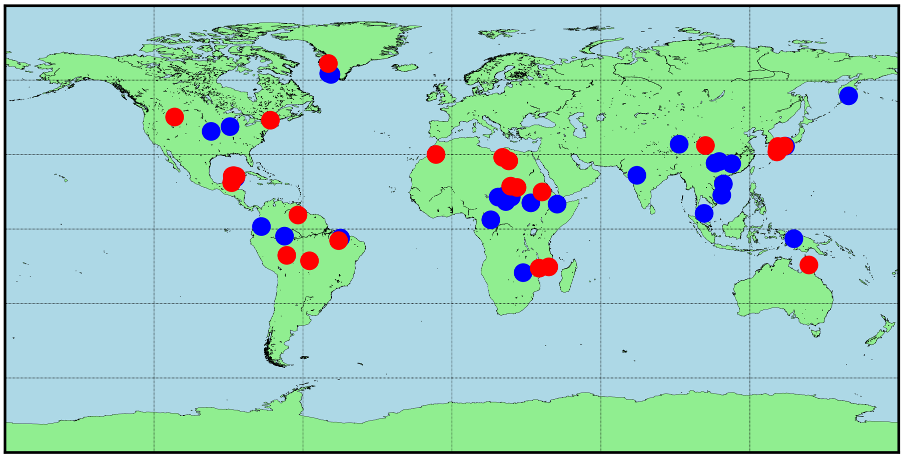

Figure 10: Plot of the positions of dragons after 2,000 days of the simulation. Blue points are from simulations where $D = 0$, red points from simulations where $D = 0.1$. For each level of D, 10 simulations were run with 3 dragons each. We see that dragons generally avoid colder regions, but there are exceptions, especially with dragons on islands and peninsulas.

support a dragon.

Location The Global Migration Model can be leveraged to suggest possible locations for a cordoned off piece of land reserved for dragons and their supporting ecosystem. Noting the blue points on Figure 10, and excluding those dragons stuck on islands in unsuitable ecosystems, we see that areas like Sub-Saharan Africa, Southeast Asia, and the Northern part of South America may be suitable climates for dragons.

Size The Regional Differential Equation Model, and, in particular, Figure 4 suggest that an ecosystem would require a minimum size of around 1,000km^2 to support a dragon. Such an ecosystem would need to be populated with 20,000 large mammals to serve as prey for the dragon. If placing all of the dragons in one nature preserve, all of these values could be tripled. When one or more dragons are introduced, the population of prey in the nature preserve will decrease, but will never be completely eliminated.

Security If a the preserve is located in a climate suitable to dragons, *i.e.* about 30°C year-round, with solar irradiance compensating when the temperature falters, security could be minimal, as the dragon would have little inclination to depart. On the other hand, if the preserve is in a highly unsuitable climate, sufficient security to keep the dragon in the preserve may well be impossible.

Support requirement The dragon, living off of its ecosystem, would require little to no support, so long as the necessary initial conditions are met for the ecosystem to survive. Thus a dragon nature preserve would be a cost-effective way to handle the arrival of the dragons.

4.4.2 Scenario 2: Dragon zoo

If dragons, contrary to the results of the Regional Differential Equation Model, are found to be wholly incompatible with any existing ecosystem, the dragons should be kept in a zoo, due to the widespread destruction the Global Migration Model shows they would cause if left to their own devices.

Location and climate control A zoo located somewhere suggested by the Global Migration Model in Figure 10 would not need to be climate controlled. Otherwise, the suitability of the climate could be regulated by heat lamps, such as is done with small pet reptiles.

Size Both the Regional Differential Equation Model and Regional Agent-Based Model suggest that dragon growth is dependent on their net energy intake. Thus, a zoo could bring a dragon to a desired size through a managed feeding strategy.

Security Dragons, growing to upwards of 500kg, would be the most massive flying animals kept in containment. This would require unprecedented measures to keep the dragon contained, especially considering that, in the wild, they naturally predate over areas of upwards of 1,000 square kilometers.

Support requirement Dragons, growing to arbitrary sizes, would need amounts of food depending on those sizes. Given the confined nature of a zoo, virtually all energy loss would come in the form of the Basal Metabolic Rate, as the dragons would be brought prey and would not need to seek it. Thus, the energy requirements of a dragon would be fully described by Equation 2. Table 3 provides calculations of yearly energy and deer at a variety of dragon sizes. The resulting values, for example, the energy requirement of 2.36×10^7 kcal/yr for a 1,000 kg dragon, roughly coincide with those of animals of similar sizes, *e.g.* an African elephant's energy requirement of 2.6×10^7 kcal/year [2]. Based on herd size information, this would require about

Table 3: Dragon energy and food requirements at different dragon sizes, assuming that dragons are in a zoo and food is brought to them

Size (kg)	Energy requirement (kcal/yr)	Food requirement (deer/yr)
100	3.04×10^6	26
250	6.88×10^6	58
500	1.27×10^7	107
1,000	2.36×10^7	199
2,500	5.32×10^7	448
50,00	9.86×10^7	828
10,000	1.83×10^8	1,534

Community Size Consider the deer industry in New Zealand. 1,800 farmers export a combined 12,911 tonnes of venison per year [1]. Given that every unit weight of venison requires approximately twice as much deer weight [9], this means that they produced 25,822 tonnes of deer, or, taking an average weight of 81kg per deer, they collectively produced about 317,000 deer [6]. That is about 176 deer per farmer. Thus, a single deer farmer would almost be able to fulfill the entire caloric needs of a 1,000kg dragon kept in a zoo, which requires 199 deer per year. For a 10,000kg dragon requiring 1,534 deer per year, 9 deer farmers would be needed to produce enough deer. Based on herd size information from New Zealand, the 9 farmers would need to collectively keep about 4,210 deer [1].

5 Model Analysis

5.1 Regional Agent-Based Model

Model Strengths and Insights The Regional Agent-based Model represents predator-prey interactions within the local ecosystem of a dragon territory to model the impact of a dragon on local prey populations as well as the potential for dragons to sustain themselves in the wild.

- The model does not set an arbitrary limit on the weight which the dragon can reach, instead having one arise naturally as a result of the increased energy expenditure which is required for flight as a dragon increases in size, ultimately leading to a caloric deficit due to a finite deer population.
- The dragon's weight increases rapidly for the first year as food is readily available and its metabolism is low, but over time the dragon's weight peaks as it reaches a weight which diminishing prey density can no longer sustain
- The dragon dies within the 10 year period measured, and at the end of the 10 years the deer population is less than 8% its initial value
- Less restrictive assumptions needed to be made in this model compared to the others as a result of our ability to simply let our algorithms run, and allowing several factors such as direction of travel or time between successive hunts to be random

Model Limitations While the model is strong for its ability to capture predator-prey interactions and dragon weight, several assumptions were made which produce limitations.

- Deer never decide to migrate as they notice their populations rapidly diminishing, and likewise as the dragon starves it never leaves its territory in search of larger populations
- As a dragon's weight decreases, it would likely go into "starvation mode" metabolically, and its hunting tactics would likely change drastically to prevent its death
- Our model for deer populations was simplistic, isolating each state of the reproductive cycles into distinct periods with no overlap, and also not accounting for predation by other animals

5.2 Regional Differential Equation Model

Model Strengths and Insights The Regional Differential Equation Model, expanding upon existing ecological models, realistically models a dragon's interaction with an ecosystem.

- The model captures a dragon's interaction with ecosystems of arbitrary sizes and prey densities.

- No assumptions are made about the maximum size a dragon can reach; a maximum size, instead, arises from the limitations of the ecosystem.
- A dragon's weight increases slowly for the first year, then its greatly accelerates as its increased size allows it to consume yet more deer.
- Rather than being such destructive forces of nature as we imagine, dragons can, given a large enough ecosystem, coexist with existing flora and fauna.
- Dragons have high caloric requirements and require many thousands of deer for sustenance.

Model Limitations While the model is strong in realistically modeling a dragon in an ecosystem, there are ways in which it is weak and areas it does not address.

- The model does not allow for multiple types of prey, who themselves are interacting with each other. That is, it does not allow for a food chain so much as a food ladder.
- The model, being complicated, contains various constants that must be assumed, researched, or approximated. While we have done this as accurately as possible, the lack of any real dragons makes the determination of these constants difficult and likely inaccurate.
- No dragon reproduction occurs. This, combined with the assumption that dragons do not decrease in weight, means that the oscillations commonly expected in predator-prey modeling do not occur.

5.3 Global Migration Model

Model Strengths and Insights The Global Migration Model reveals the migratory behavior of dragons under given levels of destructiveness.

- The model is flexible with respect to dragons' assumed impact on ecosystems, allowing it to continue to be used as understanding of dragons' impact on ecosystems increases.
- When dragons are destructive, the model shows that they will travel far over the Earth, destroying swaths of ecosystems.
- When dragons are not destructive, the model shows that they will not travel so much, and are likely to settle down in a relatively small area.
- Real-world geography and climate data are used, improving relevance compared to an abstracted model
- Using this model, we are able to consider which sites would be most suited to be dragon nature preserves by finding which locations they are least likely to leave.
- Factors influencing environmental suitability can easily be changed as understanding of dragons increases.

Model Limitations While the model is strong for its ability to capture dragon migration in a simple model grounded in the real-world, this introduces some limitations.

- Ecosystems are assumed to be completely independent. The dragon-induced collapse of an ecosystem will not affect nearby ecosystems, nor will nearby ecosystems help the destroyed one recover.
- Dragons' search area for places to migrate to is limited to a 4° by 4° square, making it hard for dragons to leave unsuitable climates like Greenland (See Figure 10).
- The model for climate suitability is simplistic, not taking into account factors like humidity (which is also important for reptiles) or dragon size.
- We do not model dragon-human interactions.

6 Conclusion

Using regional agent-based and differential equation models, we find that the effect of the introduction of dragons to ecosystems is not so dire. Combining this result with a global migration model reveals that a simple dragon management strategy is available.

With our Regional Differential Equation Model, we see that, for a wide variety of ecosystem sizes and prey densities, the ecosystem simply reaches a new steady state, with both vegetation densities and prey numbers decreasing but not being completely wiped out. However, when prey densities or ecosystem sizes are low enough, the dragon's effect on the ecosystem is catastrophic, wiping out just prey or both prey and vegetation. Parameter studies illustrate the required ecosystem size and prey density to support a dragon.

Synthesizing our Regional Differential Equation Model and our Regional Agent-Based Model, we find consistent results that ecosystems are generally resilient to dragon introduction. Based on this result, our Global Migration Model predicts that dragons, once they find the right climate, will settle and live there long-term. Hence, we recommend the creation of sufficiently large dragon nature preserves designated around where dragons naturally settle, requiring minimal intervention to keep them there and no feeding support. These would likely be in the tropical regions. The creation of dragon nature preserves in unsuitable climates, *e.g* temperate or polar climates, is not advised, as it would require much more intervention to prevent dragon migration out of the preserve.

If our regional models are found to be wrong about the effect on ecosystems, a nature preserve is infeasible, as our Global Migration Model predicts they would be very prone to leave as they damage the ecosystem. In this case, we therefore propose dragon confinement to a zoo. While this would require unprecedented security measures for the enclosure, diet could be carefully controlled to manage the size of the dragons, balancing dragon impressiveness for visitors and ease of confinement. Under any reasonable dragon size, food requirements could be met by a community of just several individuals.

Overall, the introduction of dragons to the Earth would be manageable, whether or not dragons are liable to destroy ecosystems.

7 Real-World Applications

The models designed for this paper were specifically tailored to dragons, which are large, flying reptiles. No flying reptiles currently exist, somewhat limiting the application of these models to contemporary species.

Contemporary species We believe that the two regional models, with some changes to the parameters, would be effective in modeling the interaction of large apex predators with ecosystems. Unlike many ecological models, they use predator size instead of predator number, which makes them more appropriate than traditional predator-prey models in describing the interaction of one large predator with its ecosystem. As such, they could be used to model the effect of invasive species on existing ecosystems, or could be used to determine parameter values for the predators based on observed interactions.

Prehistoric species We believe that all of the models would be highly effective in the field of paleontology. For example, pterosaurs, flying reptiles weighing 15-260kg [13], are effectively dragons that don't breathe fire. The Regional Agent-Based Model and the Regional Differential Equation Model could be used to model the interaction of pterosaurs with their environments based on estimates of their mass, and results could be compared to fossil evidence to see if our understanding of pterosaurs is accurate. The Global Migration Model could be compared with evidence about the range of pterosaurs to infer the climates that they preferred.

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Letter to George R.R. Martin

George R.R. Martin
Santa Fe, NM

Dear Mr. Martin,

Our modeling team has analyzed the outcome of introducing the three fictional dragons you describe in *A Song of Ice and Fire* to the Earth. We study prey requirements, their effect on ecosystems, their migration, and how they might best be managed by humans. The dragons we model are based on the dragons in your series. We have some results to share that might be relevant to you and some recommendations on how they may be incorporated into the plot.

Initially, we devised two models that illustrate dragon interactions with ecosystems: one based on dragons and prey as agents, and one based on differential equations.

In our differential equation model, we assume that dragon growth is ultimately limited by their energy intake vs. expenditures. This results in a natural maximum weight for dragons, based on how much they are able to hunt and how much energy the hunting takes. The greatest possible mass we were able to achieve under reasonable parameter values was just under 3,000kg. Given that limit, it is possible that the dragons kept by characters in your story, who may be fed by the characters and thus expend less energy hunting, are able to grow even more massive than the dragons that lived in the wild before the Mother of Dragons came onto the scene.

Based on this model, we also find that long-distance flights made by characters on the dragons, combined with the decreased time devoted to hunting, would, respectively, increase energy expenditures and decrease the dragons' "attack rate" (how much area they are able to hunt over in a given amount of time). Under these conditions, dragon growth would necessarily be stunted. Perhaps characters have to weigh the costs and benefits of using their dragons for transportation. One character, taking advantage of their dragon now, may end up with a much smaller and less powerful dragon than a character who leaves their dragon to grow.

We also devised a model which describes how dragons naturally move across the planet, based on climate and their ecosystems. Dragons gravitate towards regions with around 30°C, though a lower temperature can be compensated with increased sunlight. We find that, when in a good climate, dragons do not have much inclination to migrate. Thus, a character who keeps a dragon in a good climate would not have to have tamed it very well to keep it with them. On the other hand, dragons in poor climates, *e.g.* in cold, high-latitude regions, have a great inclination to migrate to better areas. Hence, somebody bringing their dragon North of The Wall would have to consider if they have tamed their dragon sufficiently well that there is little risk of it simply flying away. Such considerations could be incorporated into characters' strategic decisions in military campaigns.

We find that supplementing or fully supporting a dragon's diet, *e.g.* when no suitable hunting ground is available or when the dragon is not sufficiently tamed to allow to roam free, is feasible. Based on the model of metabolic requirements incorporated in both of our regional models, just 9 modern deer farmers, collectively providing around 1,500 deer per year, could support a 10,000kg dragon's metabolic requirements. But while that is few people, such deer production would require a large herd size of around 4,200 deer, and with the lower level of technology present in your series, more people may be required. A large herd size means a large grazing area, and it may be interesting to examine the geopolitics of seizing land as grazing area for characters' dragons. Moreover, if a dragon's personal herd of deer (or other large mammals), is attacked or stolen in some way, the characters would need to find an alternate food source to fulfill the dragon's energy requirements, introducing yet more compelling plot points.

Having completed our assessment of the ecological impact of introducing dragons to the Earth, we are confident that, consistent with your plot thusfar, dragons would not wipe out entire ecosystems. We encourage you to continue your realism in this respect. Yet we further encourage you to also incorporate some of the interesting things we have found characters may need to keep in mind when they keep dragons.

Yours sincerely,
Committee For the Incorporation of Realistic
Mathematical Models Into Works of Fantasy