The effect of the distance between two electrodes on the breakdown voltage between them

Research Question
When varying the orientation of two electrodes, what is the relationship between the distance between the electrodes and the breakdown voltage?

Aim
The aim of this experiment is to determine how changing the distance between two electrodes affects the breakdown voltage. And to, as an extension, determine how the orientation of the electrodes affects the breakdown voltage.

Personal Engagement
I build computers as a hobby and electric discharge can greatly damage components. As electric discharge can happen through gases, Paschen's law helps to know under what circumstances such discharge can happen, and what I can do to prevent it. Knowing about the impact of the orientation of the electrodes on the breakdown voltage could provide some insight on how to best package sensitive electronics.

Background
"Breakdown" occurs when an insulator becomes electrically conductive when enough voltage is applied to it. This voltage is called the insulator's "breakdown voltage", and is measured in Volts. In gases, breakdown occurs because the particles get ionized by the charge. In this experiment, the specific type of breakdown being investigated starts electric arcs. This type is started by a free electron, which is then accelerated by the electric field towards the cathode (the electrode to which electrons flow[1]) and gains enough kinetic energy to free electrons from their atoms when colliding with them, ionizing the particles. The newly freed electrons are also accelerated by the electric field, and gain enough energy to ionize more particles and release more electrons. The same happens to these created electrons. This process continues repeatedly, and releases energy in the form of heat, which turns the gas into a plasma. This accelerates the rate of ionization, as heat helps ionize particles. Like this, the arc can be sustained and electrons can flow from the anode to the cathode.

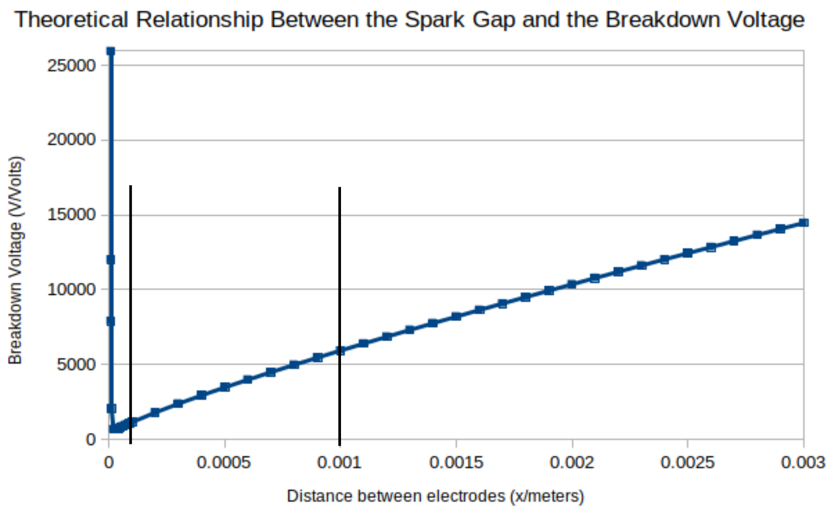
It takes a higher voltage for breakdown to occur the higher the distance or pressure is, as electrons need more energy to sustain more collisions and get to the cathode. This is reflected in Paschen's Law. Paschen's Law states that the breakdown voltage is defined by the curve:

$$V = \frac{Bpd}{\ln\left(\frac{Apd}{\ln\left(1+\frac{1}{C}\right)}\right)}$$ [2]

A, *B* and *C* are constants determined by the gas that the breakdown is happening through. In the distance and pressure ranges of this experiment, *A*, which represents the saturation ionization of the gas, is 11250 $kPa^{-1}m^{-1}$ and *B*, which represents the ionization and excitation energies of the gas, is 273750 $VkPa^{-1}m^{-1}$ [3]. *d* is the distance

1 Merriam-Webster Dictionary, s.v. "Cathode," accessed June 11, 2018, https://www.merriam-webster.com/dictionary/cathode
2 Husain, E., and R. S. Nema. "Analysis of Paschen Curves for Air, N2 and SF6 Using the Townsend Breakdown Equation." *IEEE Transactions on Electrical Insulation* EI-17, no. 4 (August 1982). doi:10.1109/tei.1982.298506.

between the electrodes, in meters. *p* is the pressure in kilopascals. *C*, the Secondary Ionization Coefficient, is 0.02 in the pressure and distance ranges of this experiment.[4]



This is a graph of the Paschen curve using the given values for *A*, *B*, and *C* and the measured *p*. The black lines show the range that this experiment focuses on, as described in the next section. More information about this graph and the data that it uses can be found in Appendix A.

Variables

The independent variable in this experiment is the distance between the electrodes. It will be measured in meters, and incremented from $1.0\times10^{-4}m$ to $1.0\times10^{-3}m$ in steps of $1.0\times10^{-4}m$. The reason for this range and increments is that using only one type of material to separate the contacts would introduce the least error into the experiment, and paper was chosen for its availability and for the ability to precisely measure its thickness. Thus the lower limit of the distance is the thickness of one sheet of paper, the upper limit of the distance is the maximum distance at which the power supply could supply the breakdown voltage, and the increments are the thickness of one sheet of paper. Paper is therefore the measuring device for the independent variable, and its thickness is measured using calipers. The dependent variable in this experiment is the breakdown voltage, which is measured in Volts using the voltage readout on the Power Supply. Its theoretical relationship to the distance is defined by the Paschen curve. It is thus hypothesized that the breakdown voltage will increase with an increase in the distance between the contacts. Three different electrode configurations are also used when obtaining the data. In one, the closest parts of the electrodes are their faces, in another, the closest parts are their edges, and in the final one, the closest parts are their corners. In the face-face configuration, because the space between the electrodes has more parts of the electrodes in closer proximity to it, the electric field in that space is stronger, and is therefore be able to accelerate the electrons to sufficient speed with lower voltages, thus decreasing the breakdown voltage. It is therefore hypothesized that the face-face configuration will have the lowest breakdown voltage, and, with the same reasoning, it follows that the corner-corner configuration will have the highest breakdown voltage.

3 Husain, E., and R. S. Nema. "Analysis of Paschen Curves for Air, N2 and SF6 Using the Townsend Breakdown Equation." *IEEE Transactions on Electrical Insulation* EI-17, no. 4 (August 1982). doi:10.1109/tei.1982.298506.

4 Ibid.

Controlled Variables

Pressure	Pressure changes are minimized by the controlled temperature, but pressure readings are taken periodically to ensure a lack of significant change. A change in pressure would affect the theoretical breakdown voltage given by Paschen's law, thus it needs to be controlled. The pressure was consistently measured at 103000Pa (Pascals).
Temperature	Controlled with an AC unit that maintains a constant temperature in the lab area. This temperature was recorded as 295.5K. Through the Ideal Gas Law, a change in temperature with a constant volume leads to a change in pressure, which affects the breakdown voltage given by Paschen's law.[5]
Resistance of circuit	The components used in the circuit, including the wires, clip component holder, resistor, power supply, electrodes, and multimeter are never changed. With the negligible resistance of the wires, multimeter, and electrodes, the resistance in the circuit is that of the resistor, which has a resistance of 10 MΩ (Ohms).

Uncontrolled Variables

Buildup on electrodes	It is possible that some positive ions, created by the breakdown, get stuck to the anode (to which they are attracted). Due to the very low current used (≤0.5mA, as limited by the resistor), it is not likely that much buildup occurs. However, it is still possible that buildup occurs, and this effectively makes the spark gap smaller and decreases the breakdown voltage as more and more trials are done. This could be limited by changing electrodes between trials or by rotating them such that other faces, edges, and corners are showing, but this would likely introduce other more significant sources of error.

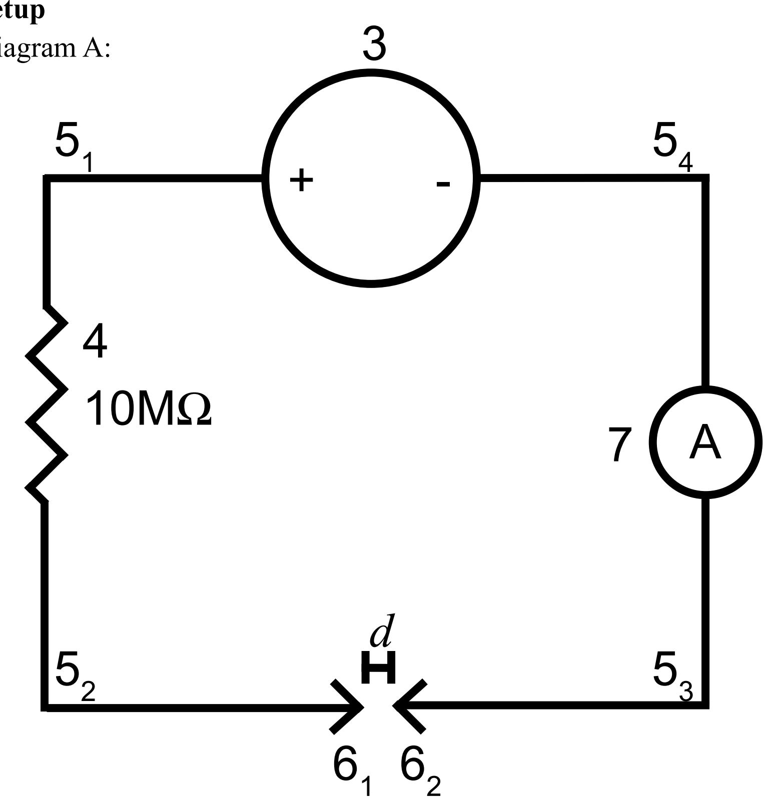
Safety

High voltages are handled in this experiment. If handled incorrectly, they can potentially be lethal. A 10 MΩ resistor is placed at the beginning of the circuit to limit the current and thus prevent the power supply's thermal trip from tripping, and provide a fail-safe if the trip does not function as intended. The only thing touched when the power supply is enabled is the knob on the power supply. When readjusting the electrodes, the power supply is off, and, when making any other changes to the circuit, the power supply is off and unplugged. All wires used are insulated. Signs are put up at any time when the experimenter is not present to warn of the experiments' danger. Protective equipment such as a lab coat and safety glasses is not necessary due to the lack of any materials—other than the power supply, the harm from which would not be mitigated by such equipment and is already mitigated in other ways—that could cause bodily harm.

5 Krönig, A. "Grundzüge einer Theorie der Gase." *Annalen der Physik und Chemie* (1856). doi:10.1002/andp.18561751008.

Setup

Diagram A:



The experimental setup can be seen in Photo A, in Appendix B.

Method

1. Arrange the circuit as seen in Diagram A, with the power supply off and unplugged and the voltage adjustment knob is turned fully to the left.
2. Measure the thickness of a ream of paper (500 sheets) using calipers, then divide the thickness by 500.
3. Use pieces of paper under part of the surface that one of the electrodes is situated on to tilt it such that, when put together, the two electrodes are completely flush with each other. If the electrodes are not completely flush with each other when put together, the trials with the faces effectively become trials with edges, and the trials with edges effectively become trials with corners.
4. Plug the power supply into mains.
5. Manually place one sheet of paper vertically in between the electrodes such that the electrodes are separated by the thin part of the paper and the bottom of the sheet is touching the surface on which the electrodes are situated.
6. Bring the electrodes together such that they are both touching the paper with a face.
7. Slowly pull the paper horizontally out from in between the electrodes, making sure to not torque them, and record the distance between the electrodes (as determined by the number of sheets)
8. Turn on the power supply.
9. Slowly turn the voltage adjustment knob on the power supply to the right.
10. Once, for at least approximately a quarter of a second, an uninterrupted and constant buzzing sound is heard or an uninterrupted, constant, and significant current is seen on the multimeter, stop turning the knob and remember the voltage.
11. Turn the knob all the way to the left.
12. Disable the power supply.

13. Record the voltage at which a constant current or a constant buzzing sound became present.
14. Repeat steps 5-13 two more times.
15. Record a pressure reading using the Absolute Pressure Temperature Sensor, ensuring that it does not deviate from the first reading taken by more than the uncertainty of the instrument (±0.5K).
16. Repeat steps 5-15, increasing the number of sheets of paper by one on every repetition until the power supply can no longer supply the necessary voltage for breakdown to occur.
17. Repeat steps 5-16 two more times, once with the electrodes' edges being the closest parts and once with the electrodes' corners being the closest parts.
18. Unplug the power supply and disassemble the setup.

Materials (images of the materials can be found in Appendix C)

	Name	Details
2	Calipers	Metal. Markings go down to millimeters, thus the uncertainty is ±0.0005m.
3	EHT (Extra High Tension) Power Unit	Produced by Philip Harris. Continuously variable from 0 to 5000 Volts. Maximum current of 0.002A.[6] Voltage uncertainty is ±10V, as determined by the last digit of the Power Unit's readout, which gives tens of volts.
4	Resistor	Resistance of 10MΩ with an uncertainty of ±0.5MΩ.
5	4 Stackable Plug Leads	Tested to have negligible resistance.
6	2 Electrodes	Metal cubes. Tested to have negligible resistance.
7	Multimeter	Produced by Extech Instruments. DC Voltage maximum of 1000V. Resistance of 0.1Ω to 20MΩ. Accuracy of ±0.5%.[7]
8	Clip Component Holder	Used to hold the resistor securely. Tested to have negligible resistance.
9	Ream of Paper	A4 copy paper. Thickness of 0.05m±0.0005m. 500 sheets. Thus each paper is 0.0001m±0.000001m.
10	2 Protractors	Plastic. Used as a surface for the electrodes.
11	Absolute Pressure Temperature Sensor	Produced by PASCO. Measures from 0 to 700kPa with an uncertainty of ±2kPa with a resolution of 0.1kPa.[8] Measures from 263K to 343K with an uncertainty of ±0.5K.

6 "EHT Power Supply." Philip Harris. Accessed May 16, 2018. https://www.philipharris.co.uk/product/power-supplies/power-supply-units/eht-power-supply/b8r02653.

7 Extech, A. FLIR Company. "Extech EX410: 8 Function Professional MultiMeter." Extech. Accessed May 16, 2018. http://www.extech.com/display/?id=14827#tab2.

8 "PASPORT Absolute Pressure/Temperature Sensor" PASCO. Accessed May 16, 2018. https://www.pasco.com/prodCatalog/PS/PS-2146_pasport-absolute-pressure-temperature-sensor/index.cfm.

Analysis

Processed Data:
Data table A

Distance Between Electrodes (x/meters)		Breakdown Voltage (V/Volts)					
		Faces		Edges		Corners	
Set value	Uncertainty	Average	Uncertainty	Average	Uncertainty	Average	Uncertainty
0.0001	1E-06	650	110	1270	100	1800	200
0.0002	2E-06	1280	100	1660	110	2310	180
0.0003	3E-06	1710	60	1970	90	2580	140
0.0004	4E-06	2320	100	2990	780	3220	160
0.0005	5E-06	2760	770	2900	110	3080	420
0.0006	6E-06	3240	460	3640	580	3640	260
0.0007	7E-06	3540	140	4110	690	3810	560
0.0008	8E-06	4270	380	4090	80	4070	90
0.0009	9E-06	4130	120	4430	190	4590	340
0.0010	1E-05	4410	120	5030	150	4930	190

This processed data was made using the raw data seen in Data Table E, in Appendix E. In the raw data, the measurement uncertainty of ±10 Volts is due to the imprecision of the Power Unit, and is determined by the last digit on the Power Unit's readout, which displays tens of Volts. The tool for measuring the distance between the electrodes, paper, is imprecise, thus the uncertainty of the distance, both in the raw data and in this processed data, is the product of the uncertainty of the thickness of one sheet of paper and the number of sheets of paper used for that trial. For example, for the row where 10 sheets were used and the distance was 0.0010m, the uncertainty is ±0.000001m×10=±0.00001m. The determination of the uncertainty of one sheet of paper is described in Materials (9).

The average was calculated by summing the results of the trials for a set and dividing that sum by the number of trials. For example, the average of Faces in the row at 0.0001m is (750V+660V+530V)/3=647V (the values for the trials used in this calculation are in Data Table E, in Appendix E). All average values are rounded to a significant thousands, hundreds, and tens digit, as those are the significant digits in the trials.

Because this experiment has significant random error, the uncertainty on the average was calculated for every row by taking the maximum value, subtracting the minimum value, and dividing that by two (effectively finding half of the range). For example, the voltage uncertainty of Faces at a distance of 0.00001m is (750V-530V)/2=±110V. All uncertainty values were rounded to a significant hundreds and tens digit, in accordance with the data.

Data Table B

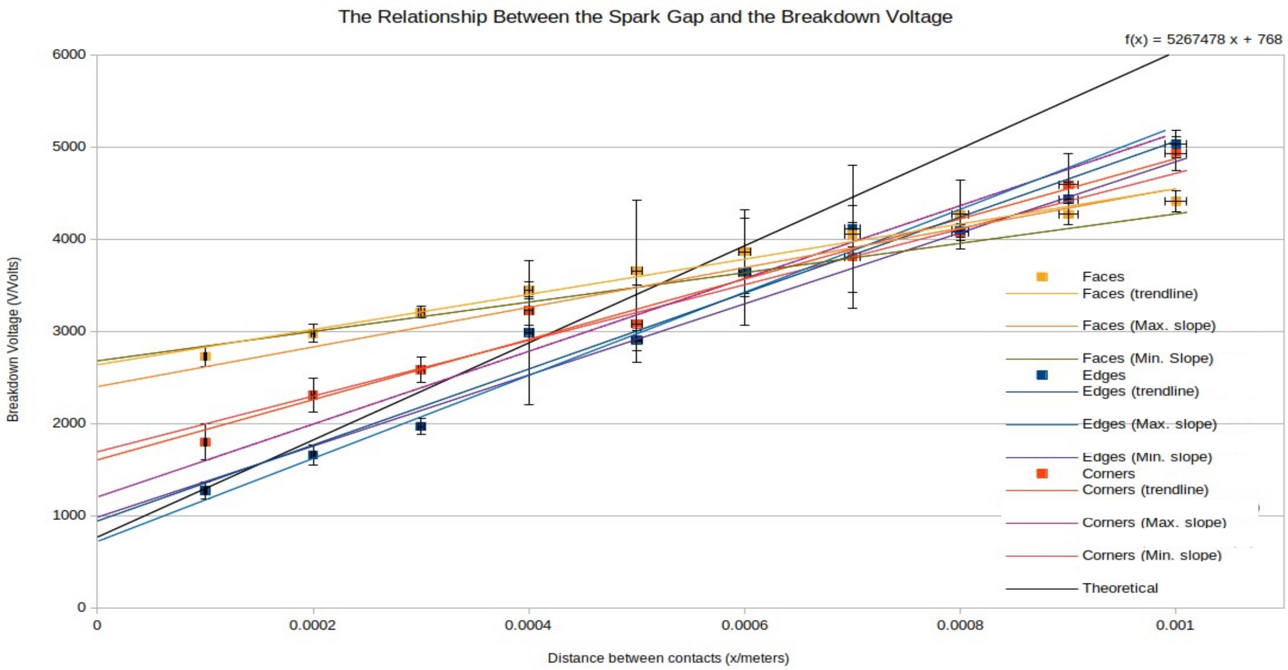
Distance Between Electrodes (m/meters)		Breakdown Voltage (V/Volts)	
Set value	Uncertainty	Average	Uncertainty
0.0001	1E-06	1240	1500
0.0002	2E-06	1750	1270
0.0003	3E-06	2090	1120
0.0004	4E-06	2840	1650
0.0005	5E-06	2910	1530
0.0006	6E-06	3510	1450
0.0007	7E-06	3820	1840
0.0008	8E-06	4150	750
0.0009	9E-06	4410	970
0.001	1E-05	4810	890

Again, the uncertainty of the distance is determined by the uncertainty by the thickness of one piece of paper. For example, for the row where 10 sheets were used and the distance was 0.0010m, the uncertainty is $\pm 0.000001m \times 10 = \pm 0.00001m$. The determination of the uncertainty of one sheet of paper is described in Materials (9).

This data table simply averages all of the trials for a particular distance. For example, for a distance of 0.0001m, the average is (750V+660V+530V+1370V+1260V+1180V+1720V+2030V+1640V)/9=1238V. All values were rounded to a significant thousands, hundreds, and tens digit, as those are the significant digits in the trials. The values for the trials in this calculation can be seen in Data Table E, in Appendix E.

The uncertainty was calculated by taking half of the range of the trials. For example, for a distance of 0.0001m, the uncertainty is (2030V-530V)/2=±750V.

Graph A



This graph relates the spark gap in meters to the breakdown voltage in Volts, using the data from Data Table A. The equations for the trendlines are as follows:

Faces: $1910000Vm^{-1}$ + 2640V.

The uncertainty of the slope is $\pm 279000Vm^{-1}$ and the uncertainty of the y-intercept is ±140V.

Edges: $4120000Vm^{-1}$ + 942V.

The uncertainty of the slope is $\pm 323000Vm^{-1}$ and the uncertainty of the y-intercept is ±263V.

Corners: $3270000Vm^{-1}$ + 1600V.

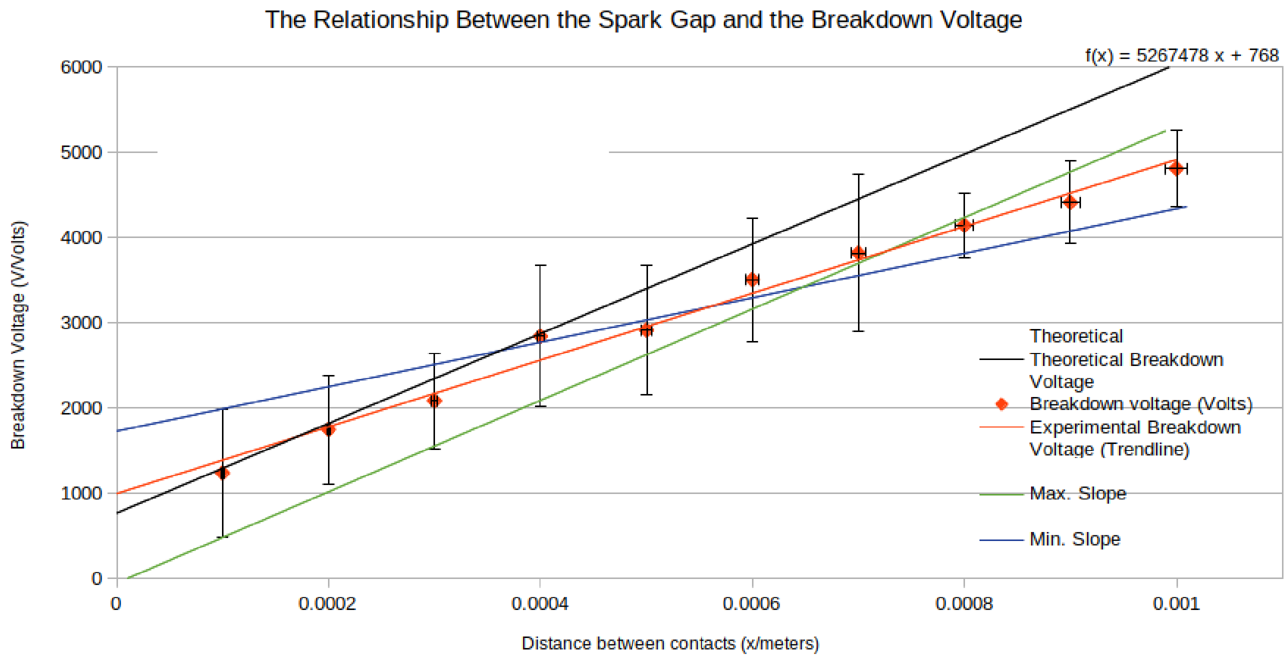
The uncertainty of the slope is $\pm 465000Vm^{-1}$ and the uncertainty of the y-intercept is ±245V.

All values were rounded to three significant figures to reflect the three significant figures of the data.

The uncertainty of the slope is half of the range of the slope in the Minimum and Maximum slope lines. For example, the Faces slope uncertainty is $(2152000-1595000)/2=\pm 279000Vm^{-1}$.

The uncertainty of the y-intercept is half of the range of the y-intercept in the Minimum and Maximum slope lines. For example, the Faces y-intercept uncertainty is (2679-2399)/2=±140V.

Graph B



This graph relates the spark gap in meters to the breakdown voltage in Volts, using the data from Data Table B.
The equations for the trendlines are as follows:
Theoretical: $5267478Vm^{-1} + 768V$.
Experimental: $3920000Vm^{-1} + 998V$.
The uncertainty of the slope is $\pm1380000Vm^{-1}$ and the uncertainty of the y-intercept is ±892V.
All values were rounded to three significant figures to reflect the three significant figures of the data.

The uncertainty of the slope is half of the range of the slope in the Minimum and Maximum slope lines. Thus, the Experimental slope uncertainty is $(5362455-260945)/2=\pm1380000Vm^{-1}$.

The uncertainty of the y-intercept is half of the range of the y-intercept in the Minimum and Maximum slope lines. Thus, the Experimental y-intercept uncertainty is $(-54-1729)/2=\pm892V$.

The X error bars in both graphs are caused by the uncertainty of the thickness of paper, the measuring device used to set the distance. The error bars get wider with higher distances because more pieces of paper are used to set those distances.

The Y error bars in both graphs are caused by the various sources of random error in the experiment. They represent the half of range of values seen in every trial. The ±10V uncertainty of the Power Unit (3) likely contributed to the very wide error bars seen at most distances in both graphs.

In both Graph A and Graph B, the y-intercepts could falsely be interpreted as stating that, at a distance of 0m, the breakdown voltage is nonzero. This is false, as, at a distance of 0, there would be no gas through which breakdown would occur. Thus, in terms of Paschen's law, the y-intercepts here are meaningless. Furthermore, as can be seen in Appendix A Graph C, the curve described by Paschen's law begins going upwards as the distance decreases, and it has no y-intercept. Given that fact, linear trendlines may seem unjustifiable. However, as can also be seen in Appendix A Graph C, it only begins curving sharply at very small distances. Further, linear trendlines generated the highest coefficients of determination—meaning that the highest proportion of the Voltage can be predicted by the Distance—when compared to logarithmic, exponential, and polynomial trendlines. Thus, within the range of this experiment, linear trendlines provide the best approximation of the curve.

Evaluation

The data supports the hypothesis that, with large spark gaps, the breakdown voltage increases with the distance. Specifically, it increases approximately linearly with the distance. This relationship is evident in that the linear trendlines generated in both Graph A and Graph B have positive slopes and coefficients of determination with values of >0.969. However, with the nonlinear nature of the Paschen curve, which can be seen in Appendix A Graph C, accurate extrapolation backwards cannot necessarily be done with this data. Despite this, as can again be seen in Appendix A Graph C, the curve stays approximately linear above the range of this experiment, thus extrapolation forwards is possible.

Graph B shows that the theoretical data is supported by the experimental data. Although not all points include the theoretical linear trendline within their uncertainty, the theoretical linear trendline, with an equation of $5267478Vm^{-1} + 768V$, is within the uncertainty for the experimental trendline, $3920000 \pm 1380000Vm^{-1}$ + $998 \pm 892V$. Thus it can be concluded that the results obtained by the experiment accurately reflect Paschen's law.

The trendline for the Edges series has the lowest coefficient of determination, showing that the linear relationship was found to be the weakest in that dataset. This suggests that there was likely significant random error in that set of trials. Some trials, like Faces Trial 2 at 0.0006m, Edges Trial 1 at 0.0004m, Edges Trial 1 at 0.0006m, Edges Trial 2 at 0.0007m, and Corners Trial 1 at 0.0007m lie far from the average values for those distances. These trials greatly affected the averages at those distances, and greatly increased the Y uncertainty of their respective points. Furthermore, any trials with a voltage greater than that which the Power Unit could supply were discarded, and this decreased the averages at those distances.

Regarding the orientation of the electrodes, no sound conclusion can be made with this data. It was hypothesized that the orientation with breakdown happening between the faces would have the lowest breakdown voltage, and that the orientation with the breakdown happening between the corners would have the highest breakdown voltage, with the orientation where breakdown happens between the edges being somewhere in between. This hypothesis is not supported by the data from 0.00001m to 0.00006m, where the Faces trials have the highest breakdown voltage and the Edges trials have the lowest. This relationship no longer applies at 0.00007m and over. Thus, no consistent data has been obtained on this. Furthermore, the orientation likely greatly impacted the various sources of error. No conclusion on the effect of the electrodes' orientation on the breakdown voltage can be made.

This investigation has several limitations. As can be seen in Graph A, some sets of trials had very large error bars, showing significant random error. One of the greatest sources of error was the method with which the electrodes were set to a certain distance. Error could have been introduced here in several ways:

- If placing the paper in between the electrodes tilted them off of the surface in any way, they would return to their original position once the paper was removed and the distance would change. This could have contributed to random error if they were inconsistently tilted, and to systematic error if they were consistently tilted. When visually inspecting a few test trials, this effect was not observed. However, it could still have occurred on a minute scale, and could be fixed by holding the electrodes more securely, possibly using clamps.

- If the paper was not held perfectly straight, and instead had a slight curve, this curve would increase the distance between the electrodes. This effect would be most prevalent in the trials with the faces. During some test trials, it did not visually appear that this issue was prevalent. However, it could still have occurred on a small scale, and one way to fix it would be to use a clamp or some other instrument that could hold the paper straight, instead of manually holding it.
- The paper could have been squished by the electrodes if they were put together with sufficient force, thus slightly decreasing the distance between the electrodes. As observed when measuring the thickness of the ream of paper, the magnitude of the change in distance increases the more piece of paper there are. Less force would be required to squish the paper with the corners or edges of the electrodes, so this may be the reason why their breakdown voltages were measured to be mostly lower than those of the trials with the faces. This could have contributed to systematic error by consistently decreasing the distance and therefore the voltage required to spark across the gap, and could have contributed to random error because different forces could have been applied when putting the electrodes together. To fix this, sheets of a firmer material, like steel, could be used.
- If there was friction between the paper and the electrodes, it could have torqued them when the paper was removed. This would have created systematic error: in the case of faces, by bringing them closer together, and, in the case of edges and corners, by separating them. To fix this, the electrodes could again be held more firmly using a device like clamps.
- In the cases of edges and corners, if they were not perfectly aligned with each other, the distance between them would increase. Because the electrodes were readjusted after every trial, this likely contributed to random error. Placing the electrodes in some kind of guide or rail would fix this.

All random error caused by these could also be mitigated by doing more trials. Systematic error could be mitigated by using the mentioned solutions.

Aside from the errors introduced by the setting the distance, error was also introduced through human error. Because of the lack of space in the lab area, the voltage readout on the Power Unit was not digitally recorded using a camera, but was remembered and manually recorded once the trial finished. It was not immediately manually recorded, because, for safety reasons, the experimenter powered off the power supply before turning away from the experiment and recording the value. This took several seconds, and some wrong values could have been recorded because of this. Furthermore, it was at the experimenter's discretion to determine when a an arc was formed. Breakdown did occur at slightly lower voltages than recorded, but this was without forming an arc. Thus the voltage was only recorded once an uninterrupted and constant buzzing sound was heard. However, in a few trials, this constant buzzing sound sometimes simply stopped after a fraction of a second, and restarted once a higher voltage was applied. Because these times were in fractions of a second, using a timer to quantitatively determine when a "constant arc" was formed would have been impossible, so the experimenter had to qualitatively determine what was a "constant arc" and what was a single breakdown. Given that the trials were done across multiple days, changes in the experimenter's qualitative assessment could have occurred and influenced the results. To fix this, a camera with a high frame-rate could be used to film the space between the contacts and the voltage readout on the power supply. However, such a camera would be unable to record the constant buzzing sound, and the experiment would have to be redesigned to use higher currents for the arc to be picked up on video. Also, only the initial voltage to start a constant arc was recorded (the voltage readout on the power supply decreased once the arc started), so the experimenter had to remember past values once it was determined that a constant arc was created.

To analyze the human error in recording the data, the last significant digit of every value was analyzed. The digit 7 was significantly under-represented in the data, only appearing 4 times. Assuming that each digit had an equal chance of appearing in the 90 trials, a binomial distribution $B(90, 0.1)$ can be used to determine the probability of a digit appearing a number of times. The probability of one digit appearing 4 or fewer times was found to be 0.047. This small probability could suggest that a bias towards or against certain digits was present, however, it does not rule out the possibility that no bias was present. It can thus be concluded that the experimenter's recollection of the values was possibly not exact, and the experimenter may have been biased towards certain numbers. In retrospect, this minor human error could be fixed by using a camera (far enough from the experiment to not be endangered by it, but close enough to see the voltage readout) to film the values and going back through the video later to take the readings.

The multimeter proved to be a difficult gauge of if or if not the breakdown voltage was reached, as its refresh rate was slow and it had a slight time delay. Thus¸ the sound created by the breakdown was used. This introduced error because, at the lower voltages, the sound was sometimes difficult to hear. Because the volume of the sound increased with the voltage, it is possible that "breakdowns" observed at lower voltages were really just an increase in volume from turning the knob. This error could be fixed by using a multimeter with a higher refresh rate and lower time delay to determine when breakdown occurred.

The instrumentation used also introduced more random error into the experiment. The Power Unit's readout had an uncertainty of ±10V, increasing the range of values in the trials. The calipers used only measured down to the millimeter, which gave the thickness of the ream of paper an uncertainty of ±0.0005m. Both of these added random error to the experiment, and this could be mitigated by using more precise instrumentation. The random error in the Voltage could be mitigated by doing more trials.

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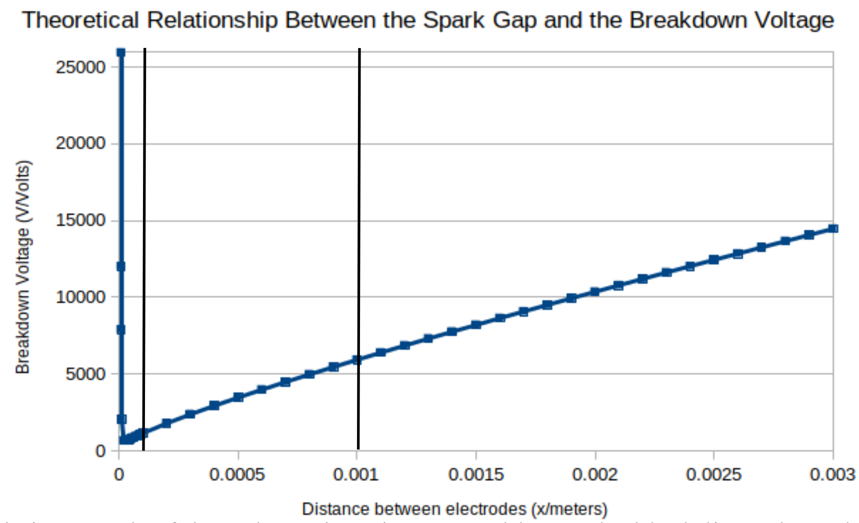
Appendix A

Theoretical Values:

Data Table C

Distance Between Electrodes (m)	Theoretical Breakdown Voltage (Volts)
0.0000088	25936
0.0000089	12020
0.0000090	7915
0.0000100	2050
0.0000200	678
0.0000300	684
0.0000400	740
0.0000500	806
0.0000600	876
0.0000700	947
0.0000800	1017
0.0000900	1086
0.0001000	1155
0.0002000	1798
0.0003000	2388
0.0004000	2945
0.0005000	3478
0.0006000	3994
0.0007000	4496
0.0008000	4987
0.0009000	5467
0.0010000	5940
0.0011000	6405
0.0012000	6864
0.0013000	7317
0.0014000	7765
0.0015000	8208
0.0016000	8647
0.0017000	9082
0.0018000	9513
0.0019000	9941
0.0020000	10365
0.0021000	10786
0.0022000	11205
0.0023000	11621
0.0024000	12034
0.0025000	12445
0.0026000	12854
0.0027000	13261
0.0028000	13665
0.0029000	14068
0.0030000	14468

Graph C

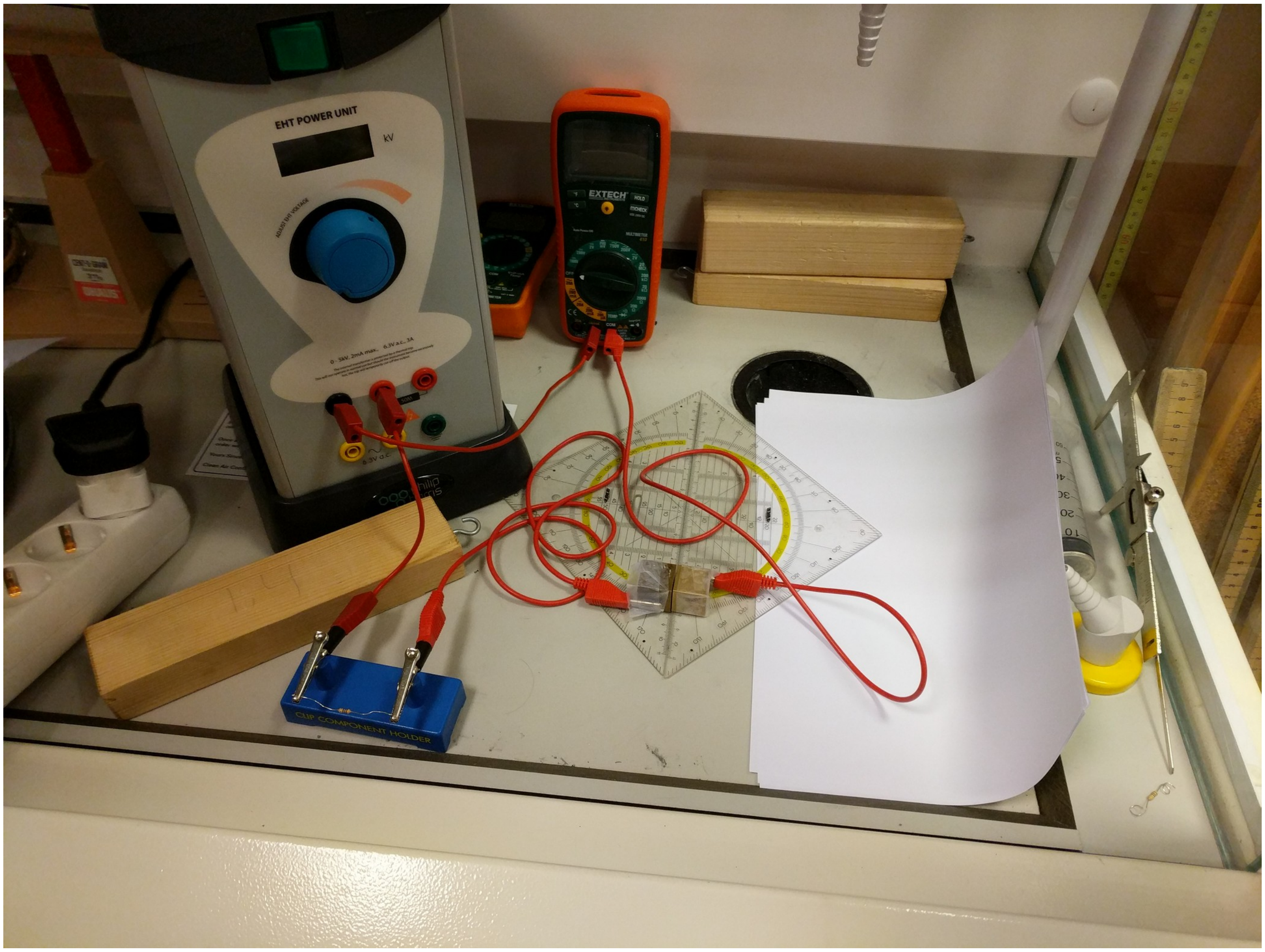


This is a graph of the values given in Data Table D. The black lines show the section of the curve that this experiment is focused on (from 0.0001m to 0.001m).

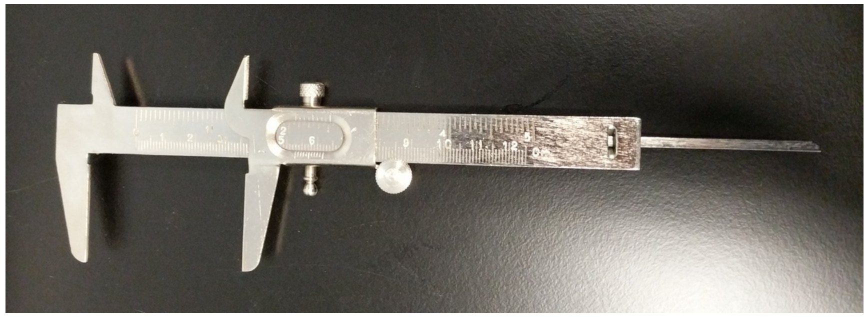
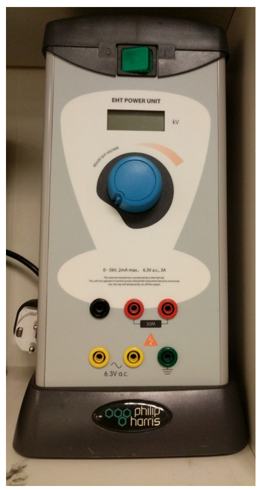
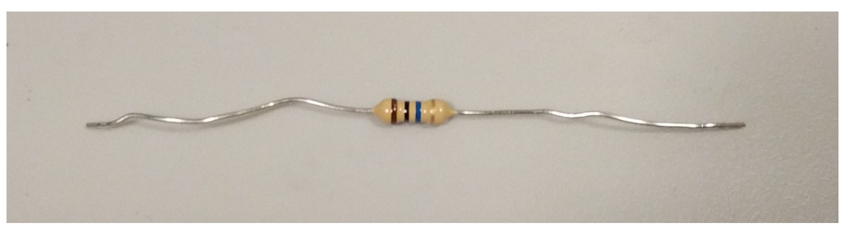
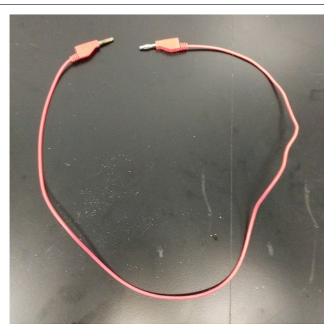
Calculations were made using the formula for the Paschen curve and the values for *A*, *B*, and *C* given in the Background section. For example, for a distance of 0.003m, the theoretical breakdown voltage is:

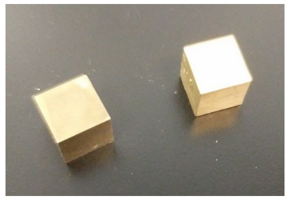
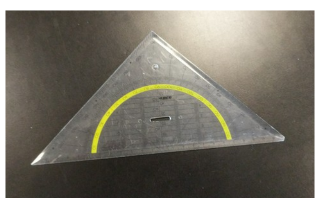
$$\frac{273.5 \times 103000 \times 0.0030000}{\ln(112.5 \times 103000 \times 0.0030000) - \ln\left(1 + \frac{1}{0.01}\right)} = 14468 \text{ Volts.}$$

Appendix B
Photo A



Appendix C
Material photos:

1	Name	Image
2	Calipers	
3	EHT (Extra High Tension) Power Unit	
4	Resistor	
5	4 Stackable Plug Leads	

6	2 Electrodes	
7	Multimeter	
8	Clip Component Holder	
9	Ream of Paper	
10	2 Protractors	

11	Absolute Pressure Temperature Sensor	

Appendix D

Data Table D—Calculation of Human Error

Last Significant Digit	Frequency	Probability of getting that frequency or lower/higher (whichever is smaller)
0	11	0.287488
1	9	0.551338
2	7	0.311487
3	13	0.112617
4	8	0.448662
5	6	0.192492
6	13	0.112617
7	4	0.046548
8	10	0.41247
9	9	0.551338

The calculations were made using a binomial distribution $B(90, 0.1)$. For example, for a frequency of 4, it can be calculated using a binomial cumulative density function with a minimum of 0 and a maximum of 4 that the probability of a frequency of 4 or less is 0.046548.

Appendix E

Raw Data:

Data Table E—The Relationship Between the Distance Between Electrodes and the Breakdown Voltage

Distance Between Electrodes (x/meters)		Breakdown Voltage (V/Volts), ±10 Volts								
		Faces			Edges			Corners		
Set Value	Uncertainty	Trial 1	Trial 2	Trial 3	Trial 1	Trial 2	Trial 3	Trial 1	Trial 2	Trial 3
0.0001	1E-06	750	660	530	1370	1260	1180	1720	2030	1640
0.0002	2E-06	1180	1370	1290	1800	1590	1580	2380	2450	2090
0.0003	3E-06	1760	1720	1640	2070	1900	1930	2480	2510	2760
0.0004	4E-06	2260	2250	2440	2340	3900	2720	3100	3160	3410
0.0005	5E-06	3640	2530	2110	3000	2780	2930	2620	3160	3460
0.0006	6E-06	2850	3760	3110	4300	3140	3490	3420	3560	3930
0.0007	7E-06	3490	3700	3430	3550	4930	3860	3090	4120	4210
0.0008	8E-06	4640	4280	3890	4000	4130	4150	4130	4130	3960
0.0009	9E-06	>5140	4240	4010	4670	4300	4320	4300	4490	4980
0.0010	1E-05	4290	4530	>5160	>5190	4880	5180	4960	5100	4730

The values with a “>” symbol indicate that the EHT Power Unit could not supply sufficient voltage to reach the breakdown voltage. The value following the ">" is the maximum voltage that the Power Unit reached in that trial before it could not supply more voltage. The trials with “>” are not included in any calculations.