A Back-Door Adjustment Formula for Soft Interventions

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Consider a causal DAG G (Pearl, 2009) and a soft intervention σ_X (as defined in Correa and Bareinboim, 2020). If $\mathbf{Z}$ satisfies the back-door criterion relative to (X, Y) (Pearl, 2009, p. 79) in both G and the interventional graph G_{σ_X}, then:

$$P(y; \sigma_X) = \sum_{\mathbf{z},x} P(y|\mathbf{z}, x; \sigma_X) P(x|\mathbf{z}; \sigma_X) P(\mathbf{z}; \sigma_X) = \sum_{\mathbf{z},x} P(y|\mathbf{z}, x) P(x|\mathbf{z}; \sigma_X) P(\mathbf{z})$$

The first equality is by conditional probability. The second follows by the σ-calculus (Correa and Bareinboim, 2020), where:

$$P(y|\mathbf{z}, x; \sigma_X) = P(y|\mathbf{z}, x)$$

by Rule 2, as $(Y \perp\!\!\!\perp X|\mathbf{Z})$ in $G_{\sigma_X \underline{X}}$ and $G_{\underline{X}}$, and:

$$P(\mathbf{z}; \sigma_X) = P(\mathbf{z})$$

By Rule 3, as $(Z \perp\!\!\!\perp X)$ in $G_{\sigma_X \overline{X}}$ and $G_{\overline{X}}$. Finally, $P(x|\mathbf{z}; \sigma_X)$ can be obtained from whatever distribution the soft intervention σ_X forces on X.

This result isn't exactly important or significant, I just thought it was neat and didn't seem to be written anywhere, so here you go. It is very reminiscent of the regular back-door adjustment formula, and in fact the regular back-door adjustment formula is a special case where σ_X is the intervention $do(x)$, and $P(x|\mathbf{z}; \sigma_X)$ becomes zero for all values of x but the one fixed by the *do*-operator. It does help solve some simple problems involving identification under soft interventions by hand. It remains to be proven that this adjustment formula holds *only if* $\mathbf{Z}$ satisfies the BDC relative to (X, Y) in both the original and interventional graphs.

References

Pearl, Judea (2009). *Causality*. Cambridge university press.

Correa, Juan and Elias Bareinboim (2020). "A calculus for stochastic interventions: Causal effect identification and surrogate experiments". In: *Proceedings of the AAAI conference on artificial intelligence*. Vol. 34. 06, pp. 10093–10100.